## Line of Sight Sensitivity Equations



## Eugene Huang

April 20, 1992

GEMINI PROJECT OFFICE 950 N. Cherry Ave. Tucson, Arizona 85719
Phone: (602) 325-9329 Fax: (602) 322-8590

## LINE OF SIGHT SENSITIVITY EQUATIONS

### 1.0 Introduction

The intent of this report is to document the line of sight sensitivity equations for a Cassegrain telescope. These equations express the image motions at the focal plane in terms of the motions of primary, secondary, and focal plane. A numerical example is given for the Gemini F/16 IR configuration. The results are checked against those obtained from Code V computer software for the same optical configuration.

### 2.0 Derivation of Equations

Figure 1 shows the location of the optical elements in a Cassegrain telescope together with the sign convention for their motions in a right hand cartesian coordinate system. The line of sight sensitivity equations are obtained from the following effects to the image motions:

1. Rotation of the primary, Rp, relative to optical axis will cause image motion of twice the angle Rp.
2. Rotation of the secondary, Rs, relative to optical axis will cause image motion of - 2 A Rs /L; where $L$ is the system focal length and $A$ is the distance between the secondary and focal plane.
3. Rotation of the focal plane will cause no image motion.
4. Translation of the focal plane, Tf , relative to optical axis will cause image motion the same amount with negative sign.
5. The rigid body rotation relative to optical axis will cause image motion of the same angle.
6. The rigid body translation will cause no image motion.

Based on the effects 1 through 4, the image motion about Y axis can be expressed by:

Here the sensitivity factors C 1 and C 2 for the translations of the primary and secondary can be obtained from the effects 5 and 6 as follows.

For a unit rigid body rotation about primary, equation (1) yields:

$$
1=2-2 \mathrm{~A} / \mathrm{L}+\mathrm{B} / \mathrm{L}+\mathrm{C} 2(\mathrm{~A}-\mathrm{B})
$$

Here B is the distance between the primary and focal plane. And this gives the sensitivity factor C2 :

$$
\mathrm{C} 2=1 / \mathrm{L}-(1-\mathrm{A} / \mathrm{L}) /(\mathrm{A}-\mathrm{B})=1 / \mathrm{L}-1 / \mathrm{Lp}
$$

Here Lp is the primary focal length which is related to the system focal length by (see figure 1) :

$$
\mathrm{L}=\mathrm{A} \mathrm{Lp} / \mathrm{H}=\mathrm{A} \mathrm{Lp} /(\mathrm{Lp}+\mathrm{B}-\mathrm{A})
$$

For a unit rigid body translation, equation (1) yieds :

$$
0=-1 / \mathrm{L}+\mathrm{C} 1+\mathrm{C} 2
$$

Hence

$$
\mathrm{C} 1=1 / \mathrm{L}-\mathrm{C} 2=1 / \mathrm{Lp}
$$

Therefore the image motion about Y axis is :

Similarly the image motion about X axis is obtained as :

### 3.0 Example

Gemini F/16 IR configuration has $\mathrm{L}=128 \mathrm{~m}, \mathrm{Lp}=14.4 \mathrm{~m}$, and $\mathrm{A}=16.539 \mathrm{~m}$. The line of sight sensitivity equations (2) and (3) give :

$$
\begin{array}{ccccc}
\mathrm{y} & \mathrm{y} & \mathrm{y} & \mathrm{x} & \mathrm{x} \\
\mathrm{Ri} & 2.0 & \mathrm{Rp} & -0.2584 & \mathrm{Rs} \\
\mathrm{R} & 0.0694 & \mathrm{Tp} & -0.0616 & \mathrm{Ts} \\
-0.0078 & \mathrm{Tf}
\end{array}
$$

or

$$
\begin{aligned}
& \begin{array}{lllllll}
\text { x y y } & \text { y } & \text { x }
\end{array} \\
& \mathrm{Ti}=\mathrm{LRi}=256.0 \mathrm{Rp}-33.078 \mathrm{Rs}+8.8888 \mathrm{Tp}-7.8888 \mathrm{Ts}-\mathrm{Tf} \\
& \begin{array}{ccccccc}
\text { y } \quad \text { x } & \text { x } & \text { y }
\end{array} \\
& \mathrm{Ti}=\mathrm{LRi}=-256.0 \mathrm{Rp}+33.078 \mathrm{Rs}+8.8888 \mathrm{Tp}-7.8888 \mathrm{Ts}-\mathrm{Tf}
\end{aligned}
$$

Here Ti is the image motion at focal plane along X axis and Ti is the image motion at focal plane along Y axis.

Table 1 shows the optical sensitivity obtained from Code V for the F/16 IR configuration. This data are consistent with the results obtained above.

## Table 1

## LINE OF SIGHT SENSITIVITY FOR GEMINI F/16 IR CONFIGURATION

| DISPLACEMENT | IMAGE MOTION AT FOCAL PLANE (mm) |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | PRIMARY |  | SECONDARY |  | FOCAL PLANE |  |
|  | x | y | x | y | x | y |
|  | Ti | Ti | Ti | Ti | Ti | Ti |
| $\mathrm{Tx}=0.025 \mathrm{~mm}$ | 0.22222 | 0.0 | - 0.19722 | 0.0 | -0.025 | 0.0 |
| $\mathrm{Ty}=0.025 \mathrm{~mm}$ | 0.0 | 0.22222 | 0.0 | - 0.19722 | 0.0 | -0.025 |
| $\mathrm{Tz}=0.025 \mathrm{~mm}$ | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| $\mathrm{Rx}=0.0005$ degree | 0.0 | -2.23402 | 0.0 | 0.28867 | 0.0 | 0.0 |
| $\mathrm{Ry}=0.0005$ degree | 2.23402 | 0.0 | - 0.28867 | 0.0 | 0.0 | 0.0 |
| $\mathrm{Rz}=0.0005$ degree | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

