8-M Telescopes
Project

# TN-O-G0005 

# Optimum Final Surface Configuration of an $8-\mathrm{m}$ Meniscus Mirror Using First and Third Order Spherical Aberrations 

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#### Abstract

A proper combination of the first order spherical aberration with the third order spherical aberration allows us to accomplish an optimum surface configuration. The optimum combination can be determined when the residual RMS surface error takes its least value from the active optics correction. The purpose of this study is to determine the optimum combination of these two spherical aberrations by parametric iterations.


## INTRODUCTION

Finite element analysis was used to demonstrate the performance of an $8-\mathrm{m}$ meniscus mirror. The current mathematic model, the one-half mirror model, comprises 318 nodal points and 294 plate bending elements. Figure 1 shows the mirror model and its support system with a total of 97 axial supports.

The active optics system was established providing an optimum set of actuator forces. The active force set can be evaluated using either Least Square Fit or Pseudo Inverse scheme in the active optics system equation defined as:

$$
\begin{equation*}
[\mathrm{A}]\{\mathrm{f}\}=\{\mathrm{b}\} \tag{1}
\end{equation*}
$$

where $[A]$ is a matrix whose components represent the displacement fields for each unit support force case, and the constraint conditions to satisfy the static equilibrium and the design requirements. [A] becomes in a short expression as:

$$
[A]=\left[\begin{array}{l}
D  \tag{2}\\
C
\end{array}\right]
$$

where [D] is the influence matrix and [C] is the constraint matrix. The influence matrix was established in terms of the optical surface displacements per unit support force based on a uniform grid of 40 by 40 over the optical surface. The constraint matrix includes the static equilibrium conditions and the design requirements in the active force set.

## SURFACE DESCRIPTIONS

An optical surface can be commonly expressed in General Polynomials as:

$$
\begin{equation*}
\mathrm{w}(\mathrm{r})=\mathrm{C}_{0}+\mathrm{C}_{1} \mathrm{r} \cos (\theta)+\mathrm{C}_{2} \mathrm{r} \sin (\theta)+\mathrm{C}_{3} \mathrm{r}^{2}+\ldots+\mathrm{C}_{36} \mathrm{r}^{12} \tag{3}
\end{equation*}
$$

where $\mathrm{C}_{0}, \ldots \mathrm{C}_{36}$ are generic coefficients of the polynomials. For example, $\mathrm{C}_{3}$ is the first order spherical aberration and $\mathrm{C}_{8}$ is the third spherical term.

A surface defined by $\mathrm{C}_{3}$ alone becomes

$$
\begin{equation*}
\mathrm{w}(\mathrm{r})=\mathrm{C}_{3} \mathrm{r}^{2} \tag{4}
\end{equation*}
$$

Similarly, a surface defined solely by $\mathrm{C}_{8}$ is

$$
\begin{equation*}
\mathrm{w}(\mathrm{r})=\mathrm{C}_{8} \mathrm{r}^{4} \tag{5}
\end{equation*}
$$

The linear combination of above two expressions with unit magnitude of the coefficients can be written as:

$$
\begin{equation*}
\mathrm{w}(\mathrm{r})=\mathrm{C}_{\mathrm{i}} \mathrm{r}^{2}+\mathrm{C}_{\mathrm{j}} \mathrm{r}^{4} \tag{6}
\end{equation*}
$$

where parameters $\mathrm{C}_{\mathrm{i}}$ and $\mathrm{C}_{\mathrm{j}}$ are corresponding to the first order spherical term and the third, respectively.
The active optics system defined by Equation (1) was utilized to evaluate the surface errors for various combinations of $\mathrm{C}_{\mathrm{i}}$ and $\mathrm{C}_{\mathrm{j}}$. The residual RMS surface errors are listed in Table 1. In the Table RMS and $\mathrm{P}-\mathrm{V}$ are in wavelength ( 1 wave $=550 \mathrm{NM}$ ) and the magnitudes were calculated based on $\mathrm{C}_{8}=$ 1.0 waves. The least RMS error was found when the ratio of $\mathrm{C}_{3}$ to $\mathrm{C}_{8}$ is 1.650 . Therefore, the normalized optimum surface figure is:

$$
\begin{equation*}
\mathrm{w}(\mathrm{r})=-1.650 \mathrm{r}^{2}+\mathrm{r}^{4} \tag{7}
\end{equation*}
$$

A scaling factor was introduced to change the reference unit from 1.0 waves to 1.0 microns for the optimized optical surface. The final optimum surface configuration is illustrated by XFRINGE as shown in Figures 2 and 3. Note that the plots were made on the Zernike surface rather than the mathematical surface. A contour plot using CODE-V was also generated to make a cross check as shown in Figure 4. Maximum and minimum values in this plot are wavefront errors at a wavelength of 550 NM.

## SUMMARY AND RESULTS

An active force set was calculated in order to minimize the optical surface defined by Equation (7) with properly scaled optimized parameters. The set of forces are required to conform to the object surface with minimum error variations. In order to perform this calculation a computer program was written with the 'llsqf' IMSL routine. The required active force distribution and summary of the results are listed in Table 2.

It was found that the RMS residual surface error for this case was 2.4 NM with a maximum required force of 14 lbs . The residual surface maps after correction are shown in Figures 5 and 6. A similar plot of CODE-V for the residual surface is also shown in Figure 7.

The RMS residual error of 2.4 NM is an interim result for the given support system as shown in Figure 1. There are many factors which impact the optical performances: the geometry of mirror, material properties of mirror blank, configuration of mirror, support system, and several other design parameters. The optical quality strongly depends upon the mirror back support system (number of supports and support pattern), especially for the active optics system.

ESO has made a similar study for the optimum ratio of the two spherical aberration (reference 3). The study observed that the ratio of 4.1 produced the optimum surface configuration with an RMS surface error of 1.0 NM .

## REFERENCES

1. Cho, M. K. and Richard, R. M., "XFRINGE -- Optical Performance Program", RMR Design Group Inc, Tucson, Arizona, 1992.
2. "CODE V ", Optical Research Associates, Pasadena, California, 1991.
3. Cui, N., Noethe, L., and Prat, S., "Axial Support System - Additional Calculations", ESO, 1992.


Figure 1. FE model and support system.


Figure 2. Contour map of the optimized surface.


Figure 3. 3-D surface map of the optimized surface.


Figure 4. Contour plot from CODE-V.


Figure 5. Contour map of residuals.


Figure 6. 3-D surface map of residuals.


Figure 7. Residual map from CODE-V.

| Ci | cj | P-V | RMS |
| :---: | :---: | :---: | :---: |
| -1.300 | 1.000 | 0.0181 | 0.00240 |
| -1.350 | 1.000 | 0.0185 | 0.00239 |
| -1.400 | 1.000 | 0.0189 | 0.00239 |
| -1.450 | 1.000 | 0.0193 | 0.00238 |
| -1.500 | 1.000 | 0.0197 | 0.00238 |
| -1.550 | 1.000 | 0.0202 | 0.00237 |
| -1. 600 | 1.000 | 0.0206 | 0.00237 |
| -1. 650 | 1.000 | 0.0210 | 0.00237 |
| -1.700 | 1.000 | 0.0214 | 0.00237 |
| -1.750 | 1.000 | 0.0218 | 0.00237 |
| -1.800 | 1.000 | 0.0223 | 0.00238 |
| -1.850 | 1.000 | 0.0227 | 0.00238 |
| -1.900 | 1.000 | 0.0231 | 0.00239 |
| -1.950 | 1.000 | 0.0235 | 0.00240 |
| -2.000 | 1.000 | 0.0239 | 0.00240 |

Table 1. Residual errors
port forces -- icase $=01$ for msc38

| supt. no | force <br> (lbs) | isupt no. | force <br> (lbs) |
| :---: | :---: | :---: | :---: |
| 1 | -7.2368 | 97 | 2.7044 |
| 2 | -3.2798 | 98 | -7.2387 |
| 3 | -0.3835 | 99 | -3.2784 |
| 4 | -0.3384 | 100 | -0.3837 |
| 5 | -3.4443 | 101 | -0.3386 |
| 6 | -7.0828 | 102 | -3.4445 |
| 7 | 4.9744 | 103 | -7.0818 |
| 8 | 5.6495 | 104 | 4.9763 |
| 9 | 4.7283 | 105 | 5.6482 |
| 10 | 4.0223 | 106 | 4.7289 |
| 11 | 3.7109 | 107 | 4.0219 |
| 12 | 4.1421 | 108 | 3.7112 |
| 13 | 4.6739 | 109 | 4.1420 |
| 14 | 5.7060 | 110 | 4.6738 |
| 15 | 4.8315 | 111 | 5.7061 |
| 16 | -1.6318 | 112 | 4.8309 |
| 17 | -1.6807 | 113 | -1.6313 |
| 18 | -2.3040 | 114 | -1.6813 |
| 19 | -1.6989 | 115 | -2.3035 |
| 20 | -1.2999 | 116 | -1.6992 |
| 21 | -1.3115 | 117 | -1.2999 |
| 22 | -1.5355 | 118 | -1.3112 |
| 23 | -1.1954 | 119 | -1.5361 |
| 24 | -1.7688 | 120 | -1.1951 |
| 25 | -2.2183 | 121 | -1.7688 |
| 26 | -1.8871 | 122 | -2.2185 |
| 27 | -1.4260 | 123 | -1.8866 |
| 28 | 4.6881 | 124 | -1.4262 |
| 29 | 3.7565 | 125 | 4.6883 |
| 30 | 5.2978 | 126 | 3.7563 |
| 31 | 4.5808 | 127 | 5.2979 |
| 32 | 4.7898 | 128 | 4.5810 |
| 33 | 4.6275 | 129 | 4.7891 |
| 34 | 3.5969 | 130 | 4.6284 |
| 35 | 5.1988 | 131 | 3.5956 |
| 36 | 3.6175 | 132 | 5.2000 |
| 37 | 4.6551 | 133 | 3.6172 |
| 38 | 4.6863 | 134 | 4.6551 |
| 39 | 4.8044 | 135 | 4.6860 |
| 40 | 5.1792 | 136 | 4.8050 |
| 41 | 4.1953 | 137 | 5.1785 |
| 42 | 4.4120 | 138 | 4.1955 |
| 43 | -10.0753 | 139 | 4.4177 |
| 44 | -7.2571 | 140 | -10.1755 |
| 45 | -10.7975 | 141 | -7.2571 |
| 46 | -9.0086 | 142 | -10.7976 |
| 47 | -9.9139 | 143 | -9.0088 |
| 48 | -8-8884 | 144 | -9.9139 |
| 49 | -10.6384 | 145 | -8.8881 |
| 50 | -7.4187 | 146 | -10.6386 |
| 51 | -9.9691 | 147 | -7.4183 |
| 52 | -9.9952 | 148 | -9.9688 |
| 53 | -7.4011 | 149 | -9.9963 |
| 54 | -10.6367 | 150 | -7.4007 |
| 55 | -8.9786 | 151 | -10.6368 |
| 56 | -9.8387 | 152 | -8.9780 |


| 57 | -9.3080 | 153 | -9.8396 |
| :---: | :---: | :---: | :---: |
| 58 | -10.4639 | 154 | -9.3073 |
| 59 | -8.1179 | 155 | -10.4638 |
| 60 | -9.6346 | 156 | -8.1184 |
| 61 | 2.3476 | 157 | -9.6338 |
| 62 | -0.1318 | 158 | 0.1317 |
| 63 | 10.8044 | 159 | 10.8042 |
| 64 | -10.6045 | 160 | -10.6043 |
| 65 | 13.6609 | 161 | 13.6610 |
| 66 | -5.9989 | 162 | -5.9995 |
| 67 | 11.5895 | 163 | 11.5907 |
| 68 | -8.4644 | 164 | -8.4659 |
| 69 | 13.1165 | 165 | 13.1181 |
| 70 | -7.5638 | 166 | -7.5650 |
| 71 | 13.7161 | 167 | 13.7174 |
| 72 | -9.4009 | 168 | -9.4029 |
| 73 | 12.1728 | 169 | 12.1757 |
| 74 | -6.0916 | 170 | -6.0949 |
| 75 | 13.7061 | 171 | 13.7088 |
| 76 | -10.6497 | 172 | -10.6516 |
| 77 | 11.0610 | 173 | 11.0616 |
| 78 | -0.7123 | 174 | -0.7124 |
| 79 | 3.4991 | 175 | 3.4992 |
| 80 | -0.8788 | 176 | -0.8785 |
| 81 | 11.3952 | 177 | 11.3940 |
| 82 | -10.9947 | 178 | -10.9919 |
| 83 | 13.8777 | 179 | 13.8746 |
| 84 | -6.1406 | 180 | -6.1386 |
| 85 | 12.1866 | 181 | 13.1856 |
| 86 | -9.1917 | 182 | -9.1915 |
| 87 | 13.0898 | 183 | 13.0895 |
| 88 | -6.2675 | 184 | -6.2673 |
| 89 | 11.1808 | 185 | 11.1819 |
| 90 | -5.9092 | 186 | -5.9118 |
| 91 | 8.1089 | 187 | 8.1111 |
| 92 | -1.2755 | 188 | -1.2765 |
| 93 | 9.3083 | 189 | 8.3083 |
| 94 | -5.9042 | 190 | -5.9038 |
| 95 | 8.5440 | 191 | 8.5440 |
| 96 | 0.3258 | 192 | 0.3255 |


| summary of the axial force set (lbs): |  |  |  |
| ---: | ---: | ---: | ---: |
| max | min | $\mathrm{p}-\mathrm{v}$ | rms |
| 13.8777 | -10.9947 | 24.8724 | 7.4956 |

summary of object displacement field (waves):

| $\max$ | min | $\mathrm{p}-\mathrm{v}$ | rms |
| ---: | ---: | ---: | ---: |
| -0.0920 | -1.2374 | 1.1454 | 0.3439 |

summary of residual displacements (waves) :

| $\max$ | min | $\mathrm{p}-\mathrm{v}$ | rms |
| ---: | ---: | ---: | ---: |
| .0112 | -0.0270 | 0.0382 | 0.0043 |

