Gemini Mount Control System Report

Prediction Of Servo Error Using Simulink Model

Gemini

9 July 1996

MCSJDW10 (Issue 3) - Describes the process of estimating the performance of the main axis servos using the non-linear Simulink model as developed by Mike Burns. These errors are then compared with the error specification as agreed with the IGPO.

1. INTRODUCTION

The errors from the servo system are considered as the superposition of three things:

i. Linear Servo Error, E_s. Error due to the bandwidth (or lack of it) of the servo. This can be found by performing a frequency response of servo error with respect to input.
ii. Encoder Error, E_e. The difference between the actual position and the position indicated by the encoders. This has been found by experimentation - see MCSJDW11.
iii. Telescope Error, E_t. Error that is caused by non-linear effects that are outside the bandwidth of the telescope. This is found by performing an FFT upon the servo error signal from a non-linear simulation.

Figure 1 shows where these errors occur within the telescope servo loop.

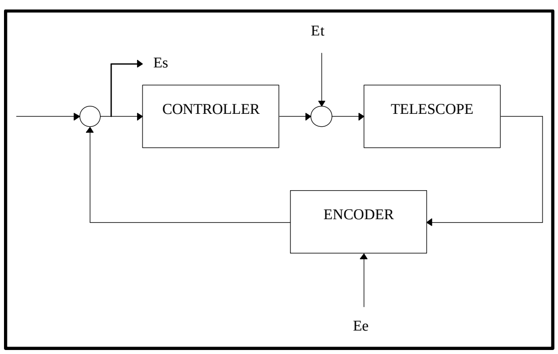

Figure 1 - Location Of The Various Error Signals

To predict the errors from the telescope the reduced version of Mike Burn's non-linear model has been used. This consists of two models, one for each axis. See Appendix C of the MCS CSDD for more details. Note that these models do not include the effect of the telescope pier - this allows the elevation axis model to achieve a much larger bandwidth than is possible in reality. A linear version of each model, which had all the non-linear effects removed, was also used.

A model of the PMAC card has been used to act as the position controller. This is shown in Figure 2.

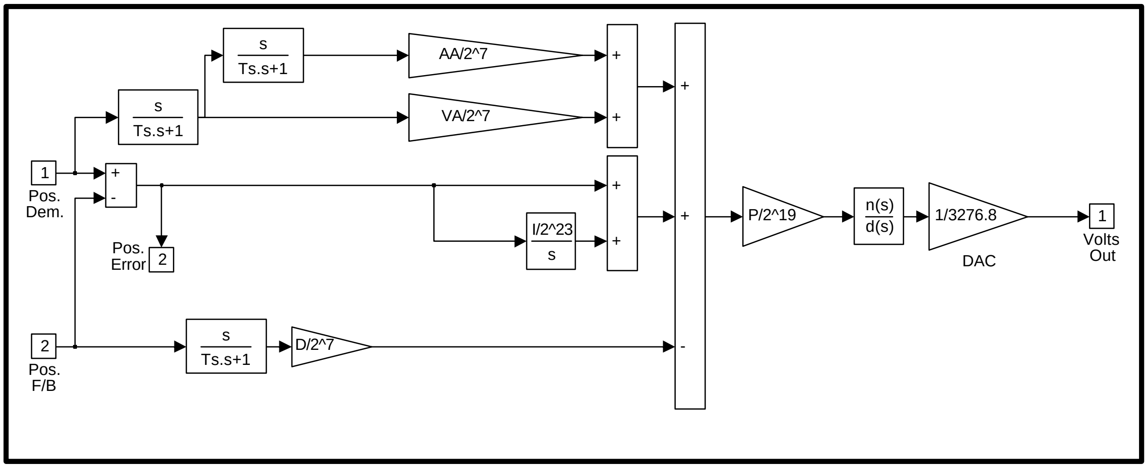

Figure 2 - Linear, Continuous Time PMAC Model

2. AZIMUTH AXIS

The controller was optimised to give a large bandwidth. The following PMAC parameters, with reference to Figure 2, were chosen:

P = 188365
I = 6400000
D = 3
VA = 17.41
AA = 0.448

These give a bandwidth of about 6.3 rad/sec (1 Hz).

2.1 Linear Servo Error

Taken from the linearised model using the ‘linmod2’ and ‘bode’ functions in MATLAB. The response is as shown in Figure 3.

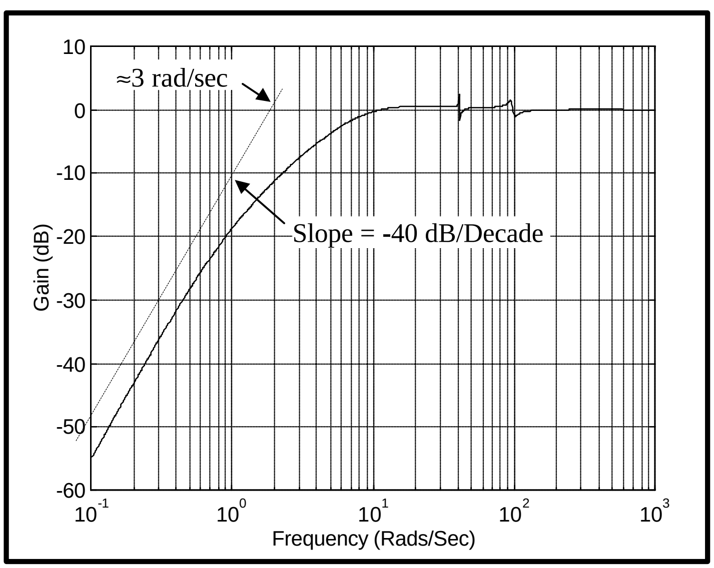

Figure 3 - Azimuth Linear Error Bandwidth

Figure 3 shows that the linear servo error is dependant upon frequency. In fact, if we approximate the error bandwidth to a cut-off frequency (*BW* in rad/sec) and a slope (*Sl* in dB/decade), for a given sinusoid ($P_{eq} \sin W_{eq}t$), the error will be given by:

$$Error = P_{eq} 10^{\frac{Sl \times \log_{10}\left(\frac{BW}{W_{eq}}\right)}{20}}$$

This gives the azimuth axis error, the error upon the sky is modified by a projection factor which is the cosine of elevation (E), thus sky error is given by:

$$SkyError = Error \times \cos E$$

The parameters of the sinusoid (P_{eq} & W_{eq}) will be dependant upon the azimuth velocity and acceleration, thus:

$$P_{eq} = \frac{V_{\max}{}^2}{A_{\max}},$$

$$W_{eq} = \frac{A_{\max}}{V_{\max}}$$

For a given elevation, the values of $V_{\max}$ and $A_{\max}$ can be found using the equations as described by Pat Wallace in "Geometrical Transformations for the Gemini Telescopes" (TCS/PTW/1.5).

Using all this, we can calculate the sky error for stellar tracking over a range of elevations. Figure 4 shows the result of doing this for latitude = 19.8° (Mauna Kea), *BW* = 3 rad/sec and *Sl* = -40 dB/decade (See Figure 3).

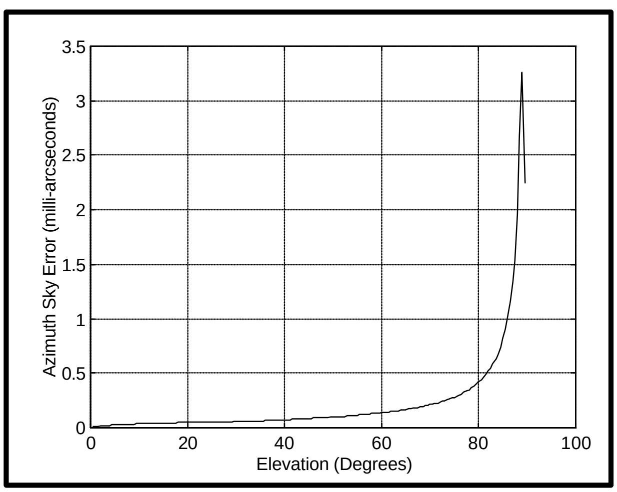

Figure 4 - Sky Error Due To Azimuth Linear Servo Error

From this, we can see that the maximum error (≈3.3 milli-arcseconds) occurs at approximately 88.9° elevation. The W_{eq} corresponding to this is 1.4×10^{-3} rad/sec - so small it can be considered a DC error.

Note that the equivalent to this maximum error on the azimuth axis is 172 milli-arcseconds. This is very large, but the errors only occur at such magnitudes at a very few values of elevation, at most elevations (say 80° and below) the azimuth error is less than 3 milli-arcseconds.

2.2 Encoder Errors

From encoder tests carried out in the lab, see MCSJDW11, the encoder error is expected to be around 20 milli-arcseconds RMS at a velocity dependant frequency. The profile of this disturbance then, is given by the closed loop servo bandwidth, i.e. at frequencies of 1 Hz and lower the error will be 20 milli-arcseconds. At higher frequencies the error will be attenuated according to the roll-off of the frequency response.

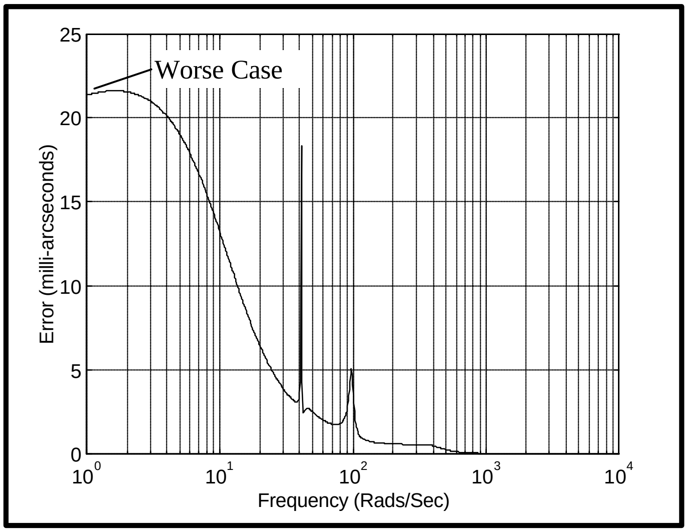

Figure 5 - Expected Encoder Errors With Respect To Frequency

Note the nasty spikes in the error at 41 and 97 rad/sec (6.5 and 15.4 Hz respectively), which will be due to structural resonance.

The maximum encoder error would occur when the axis velocity caused the error frequency to be 1.6 rad/sec (0.255 Hz). The velocity at which this occurs will depend upon which tape encoder is chosen (0.44 arcseconds/second for Heidenhain and 10.95 arcminutes/second for Inductosyn). The amplitude of this error is 21.7 milli-arcseconds.

2.3 Telescope Errors

The non-linear model was simulated for 20 seconds with the various inputs listed below. An FFT was then performed upon the servo error signal. Only the last ten seconds worth of data was used for the FFT to ignore the settling part of the response. The inputs used were:

1 Encoder Count (E.C.) Step
6000 E.C. Step (30 arcs)
720000 E.C. Step (1°)
3000 E.C./sec Ramp (15 arcsec/sec)
360000 E.C./sec Ramp (0.5°/sec)

There was always an error at 6.6 Hz, though at different amplitudes depending on the input. An error at 9.3 Hz appeared in the 360000 E.C./sec ramp case. An error at 15.1 Hz appeared in the 1 E.C. step case. The summary of the worst case results is:

6.6 Hz   13 E.C. (65 milli-arcseconds) peak.

9.3 Hz 5 E.C. (25 milli-arcseconds) peak.
15.1 Hz 3 E.C. (15 milli-arcseconds) peak.

The 6.6 and 15.1 Hz errors will be due to excitation of structural resonance. The 9.3 Hz error, does not correspond to a structural resonance so must be due to some non-linear process.

2.4 Total Errors

To super impose the three types of error discussed above, the worse case frequency components must be combined in a root sum square (RSS). This is shown in Table 1.

Frequency (Hz)	RMS Amplitude (milli-arcseconds)	Cause
D.C.	132*	Linear Servo Errors
0.255 Hz	21.7	Encoder Error
6.6 Hz	46	Telescope Errors
9.3 Hz	17.7	
15.1 Hz	10.6	
TOTAL (RSS)	55 / 134*	

Table 1 - Worse Case Azimuth Error, Without Tip/Tilt Correction

* Note that it is unfair to use the large value for linear servo error since it only occurs at a small number of elevations, at which the azimuth errors contribute little to the overall sky error anyway. At most elevations, the azimuth linear servo error is negligible.

2.5 Effect Of Tip/Tilt Correction

The effect of the active tip/tilt secondary loop can be modelled by a second order servo loop, with a bandwidth of 25 Hz and a damping factor of 0.707. The transfer function of such a loop is:

$$G(s) = \frac{24649}{s^2 + 222s + 24649}$$

This loop uses an actual star for position feedback, therefore all the errors discussed previously will be act as disturbances to the tip/tilt loop. The disturbance rejection response of the tip/tilt loop is given by the error transfer function:

$$E(s) = 1 - G(s) = \frac{s^2 + 222s}{s^2 + 222s + 24649}$$

The frequency response of this transfer function is the disturbance rejection response and it is shown in Figure 6. The effect of the azimuth encoder errors and telescope errors after tip/tilt correction shown in Figure 7. The effect of the linear servo error, because of its low frequency, is negligible after tip/tilt correction.

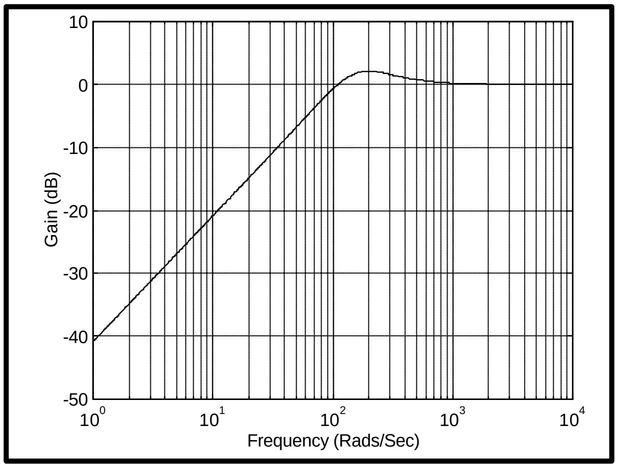

Figure 6 - Disturbance Rejection Response Of Secondary Tip/Tilt Loop

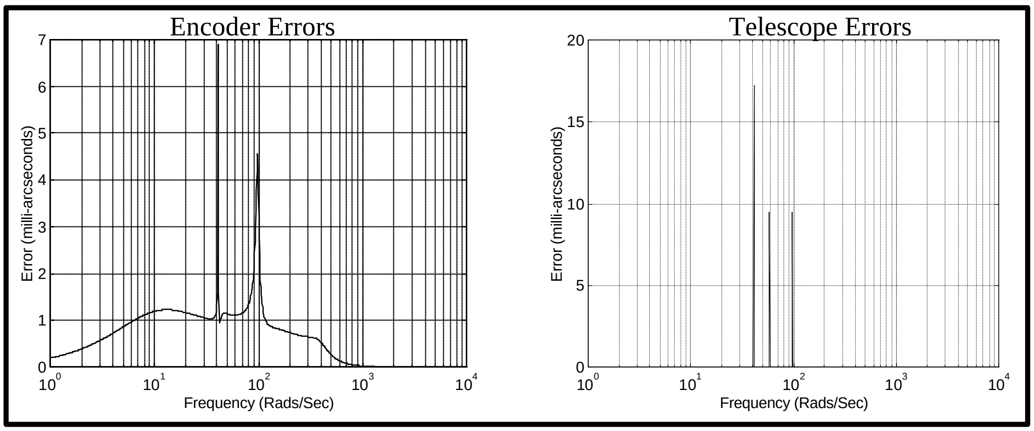

Figure 7 - Effect Of Tip/Tilt Upon Azimuth Errors

Note that the maximum encoder error now occurs at 6.6 Hz. This corresponds to an axis velocity of 11.35 arcseconds/second for the Heidenhain tape or 4.73 arcminutes/second for the Inductosyn tape.

Table 2 shows the combined effect of all azimuth errors after tip/tilt correction.

Frequency (Hz)	RMS Amplitude (milli-arcseconds)	Cause
6.6 Hz	6.9	Encoder Error
6.6 Hz	17.3	Telescope Errors
9.3 Hz	9.6	
15.1 Hz	9.6	
TOTAL (RSS)	≈23	

Table 2 - Worse Case Azimuth Error, After Tip/Tilt Correction

3. ELEVATION AXIS

The controller was optimised to give a bandwidth of about 1 Hz. The following PMAC parameters, with reference to Figure 2, were chosen:

P = 9000

I = 7600000
D = 3
VA = 0
AA = 0

These give a bandwidth of about 6 rad/sec (≈1 Hz).

3.1 Linear Servo Error

Taken from the linearised model using the 'linmod2' and 'bode' functions in MATLAB. The response is as shown in Figure 8.

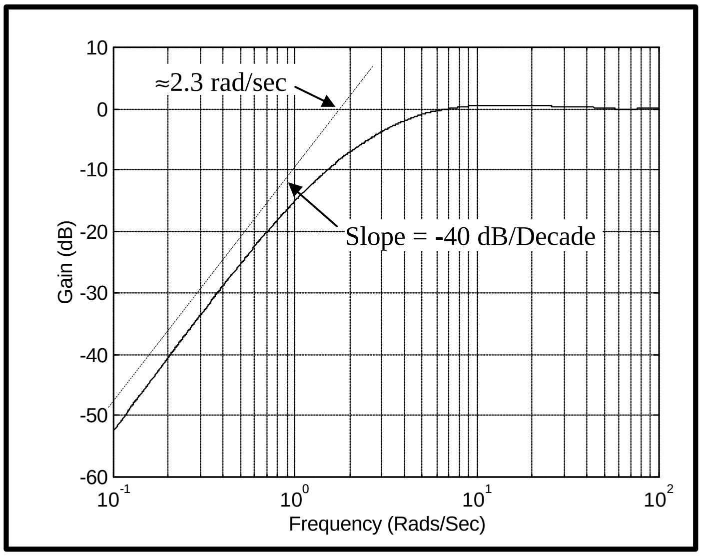

Figure 8 - Elevation Linear Error Bandwidth

Unlike azimuth, there is no sky projection factor to be taken into account for elevation. The maximum sky error, is therefore equal to the maximum axis error. Using Pat Wallace's equations, it can be shown that the maximum velocity and acceleration will occur at the closest zenith point (zenith distance = 0.5°). The values for Mauna Kea are:

$$V_{max} = 3.9\text{x}10^{-3}\ °/\text{sec}$$
$$A_{max} = 3.1\text{x}10^{-5}\ °/\text{sec}^2$$

Using the equations derived above for the azimuth axis, this values give a maximum equivalent sinusoid 0.5 sin $(8\text{x}10^{-3})t$. Using BW = 2.3 rad/sec and Sl = -40 dB/decade (See Figure 8), we get a maximum error of 22 milli-arcseconds at a frequency of 1.3 mHz (effectively D.C.).

3.2 Encoder Errors

As with the Azimuth axis, the error due to the encoder is expected to be around 20 milli-arcseconds RMS at a velocity dependant frequency or frequencies. Figure 9 shows this applied to the elevation closed loop frequency response.

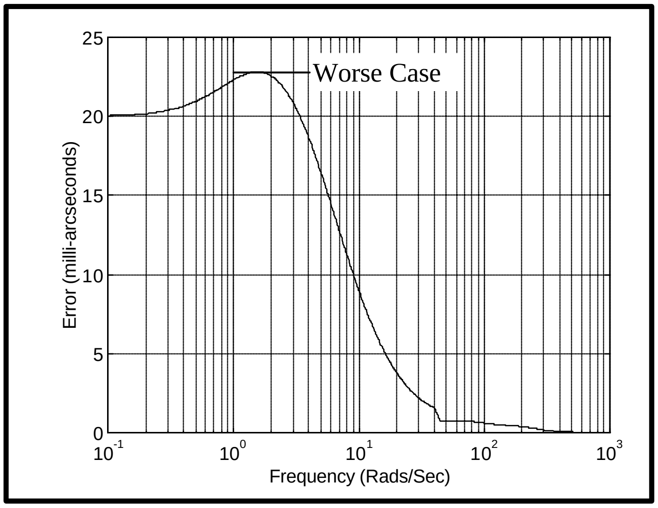

Figure 9 - Expected Encoder Errors With Respect To Frequency

Note the absence of any sensitive resonance such as those seen with the azimuth axis. This is probably because the natural modes of the telescope tube are at higher frequencies than those of the azimuth axis and are completely damped out by the servo bandwidth.

The maximum encoder error would occur when the axis velocity caused the error frequency to be 1.52 rad/sec (0.242 Hz). This corresponds to an axis velocity of 0.5 arcseconds/second for Heidenhain and 12.6 arcseconds/second for Inductosyn). The amplitude of this error is 22.9 milli-arcseconds.

3.3 Telescope Errors

The same tests were applied to the elevation axis as the azimuth axis. There were no significant errors and again, this is put down to the high (as compared to servo bandwidth) frequency modes of the elevation axis.

3.4 Total Errors

As in Section 2.4, to super impose the errors discussed above, the worse case frequency components must be combined in a root sum square (RSS). This is shown in Table 3.

Frequency (Hz)	RMS Amplitude (milli-arcseconds)	Cause
D.C.	15.55	Linear Servo Errors
0.242 Hz	22.9	Encoder Error
TOTAL (RSS)	≈27.7	

Table 3 - Worse Case Elevation Error, Without Tip/Tilt Correction

3.5 Effect Of Tip/Tilt Correction

The elevation errors, can be applied to the disturbance rejection response as shown in Figure 6. The effect of the encoder error after tip/tilt correction is shown in Figure 10. The effect of the linear servo error, because of its low frequency, is negligible after tip/tilt correction.

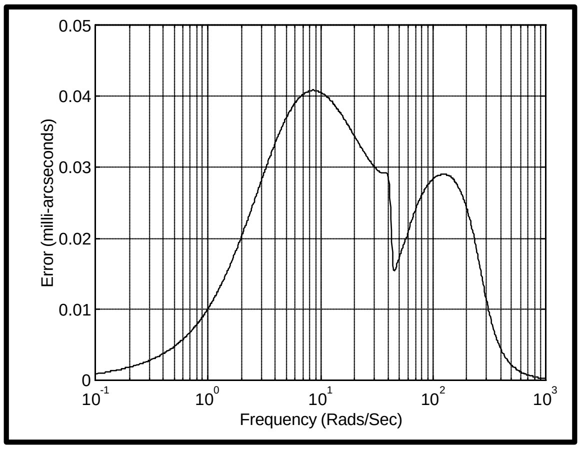

Figure 10 - Effect Of Tip/Tilt Upon Elevation Encoder Errors

It can be seen that, after tip/tilt, the combined effect of all elevation errors is negligible, i.e. less than 0.05 milli-arcseconds RMS.

4. SERVO SPECIFICATION

The error budget agreed with the IGPO is 20 milli-arcseconds RMS, on axis, at zenith. This increases with zenith distance, *Z*, according to the following equation:

$$E = 0.02 \times \sqrt{1 + \sin^2(Z)}$$

Where *E* is the permitted error, on axis, in arcseconds.

The on axis error is calculated from individual axis errors by using the following equation:

$$E_T = \sqrt{{E_E}^2 + \left[E_A \times \sin(Z)\right]^2}$$

Where: E_T is total on axis error.
E_A is total azimuth error.
E_E is total elevation error.

Table 4 summarises the results of the previous sections and shows maximum RMS error expected from each axis.

RMS milli-arcseconds	Without Tip/Tilt	With Tip/Tilt
Azimuth	55	23
Elevation	27.7	0.05

Table 4 - Summary Of Error Predictions

Figure 11 shows how the predicted on axis errors, with and without tip/tilt, compare to the current servo specification.

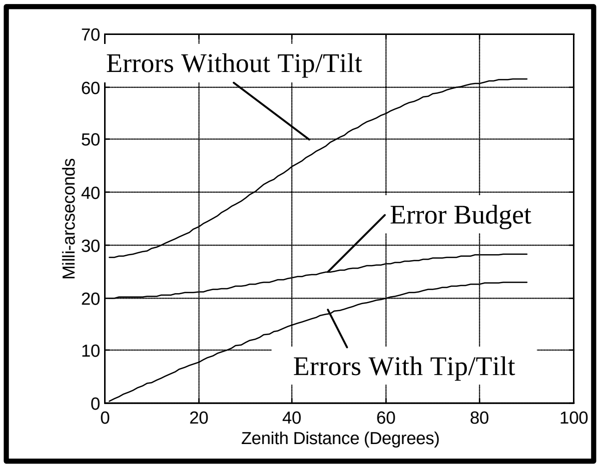

Figure 11 - On Axis Errors For The Range Of Zenith Distance