To: Rick McGonegal

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Date: June 2, 1994

Subject: Windshake vs. Sample Rate and Centroid Error vs. Sample Rate for Tip-Tilt Using an Off-Axis Guide Star (updated)

References:

[1] Ulich, Bobby L. "Overview of Acquisition, Tracking and Pointing System Technology" Proceedings SPIE 1988. Acquisition, Tracking and Pointing II. pp40-63.

[2] Burns, Mike "SNR vs. Sample Rate for Tip-Tilt Using an Off-Axis Guide Star" Technical Note, Gemini 8-M Telescopes Project. August 1993.

[3] Burns, Mike "Windshake vs. Sample Rate and Centroid Error vs. Sample Rate for Tip-Tilt Using an Off-Axis Guide Star", Technical Note, Gemini 8-M Telescopes Project, August 1993.

1. SUMMARY

Using the 3.5 arcminute diameter science field to find a bright star with which to compensate for windshake meets the error budget over most of the sky with a sampling rate of 200Hz. Only the cases pointed into the wind and near the horizon (labeled TY30 and TY90 in the text) are not met with a 3.5 arcminute diameter field. In these cases, even an 11.5 arcminute diameter field does not make the error budget because the error is dominated by the windshake and only little affected by the centroid error.

From [2] the 3.5 arcminute diameter field provides neither attenuation nor amplification of atmospheric induced tip-tilt error. More recently, information from Brent Ellerbroek of Starfire Labs seems to indicate that a 6.0 arcminute diameter field without suffering an increase in the atmospheric induced tip-tilt error. This larger field will guarantee a brighter guide star and reduce the centroid measurement error. The centroid measurement error is a significant contributor to image smear over much of the sky. Unfortunately, the TY30 and TY45 cases are dominated by windshake and will not be significantly helped by the reduction in centroid error.

2. INTRODUCTION

The results presented here are updated from Ref [3].

Building on the earlier note of SNR vs. Sample Rate, this note computes the disturbance shake of the telescope and the RMS centroid error vs. sample sate and root sum squares (RSS) them to calculate a net RMS error. Note that the errors are ignored which result from atmospheric decorrelation due to spatial difference between guide star and science object.

3. TIP-TILT ERROR SOURCES

There are two types of image smear error sources to be considered, disturbances and measurement noise. Figure 1 helps distinguish between these. The three disturbances as the far left of figure 1 can be though of as commanded positions for the secondary mirror. These commands are compared to the actual secondary position to get the true tracking error. The true tracking error is corrupted by additive measurement noise to give the measured tracking error. The controller acts upon the measured tracking error to supply a torque through the actuators to the secondary mirror, changing its position and closing the loop. It can be seen that for a purely open loop case, if the controller were removed, the only errors that would be noticed would be the disturbance errors at the far left of figure 1. The measurement errors only affect tracking when the loop is closed.

The following sections describe in some detail the individual disturbances (windshake, enclosure shake and nonlinear control system shake) and the measurement errors (centroid noise and off-axis guiding error).

3.1 Windshake

The RMS windshake was calculated by using a finite element analysis (FEA) to produce a spectrum of image motion due to the direct effect of wind upon the telescope structure. The spectrum of figure 2 is in the form of a power spectral density, radians^2/Hz vs. frequency. Integrating the spectrum over frequency gives angular noise power. Taking the square root then gives the RMS image centroid motion due to windshake without any correction by the servo loop.

If the windshake spectrum is passed through a filter, representing the servo system, then a spectrum "after compensation" results. This latter spectrum will typically have less energy in the lower frequencies because the servo system is capable of tracking these slower moving disturbances. The "after compensation" spectrum may be integrated and the square root gives the compensated windshake RMS image centroid motion.

The servo in this case is assumed to be a sampled data system with bandwidth equal to 1/8th of the sampling frequency. The servo loop is 4th order and includes an anti-aliasing filter to prevent the noise associated with sampling from being "folded-back" into the filter. There is also a lead-lag filter to improve the phase margin near the gain-crossover frequency. Higher bandwidth is desirable, but is limited to sampling/8 by the restriction of maintaining 60 degrees of phase margin.

The motion of the telescope tube due to wind is dependent upon the altitude of the tube above the horizon and upon the direction of wind relative to the telescope. Many different wind cases are listed in Table 1 of the results section below. The cases labeled TY# are for the telescope pointed into the wind at an angle # degrees above the horizon. The cases labeled TX# are for the telescope pointed across the wind at an angle # degrees above the horizon. The into the wind cases are the most difficult for which to meet the error budget.

3.2 Enclosure Induced Shake

Another effect associated with wind is the motion of the telescope enclosure which is transmitted through the pier and then to the telescope tube, causing image smear. Figure 3 shows the uncompensated spectrum for this effect, which is expected to be nearly independent of wind direction.

3.3 Nonlinear Control System Induced Shake

The control system will be imperfect and will contribute to error in the image plane. Among the nonlinear errors are altitude and azimuth axis encoder quantizations, bearing frictions, motor torque noise (both cogging and torque constant variation), tachometer ripple, motor D/A conversion error, and drive wheel eccentricities. Figure 4 shows the uncompensated spectrum for these in both the x and y image directions for a typical case.

3.4 Centroiding Measurement Noise

The wavefront sensor can not perfectly compute the image centroid. This section quantifies the centroid errors which are plotted in figure 5 for a 200Hz sampling rate.

From ref [1] the full width half max error is given by

fwhm = 0.995 * A^(3/5) * lambda / R0

where

A = 1

lambda = 0.7e-6 m = wavelength of interest
R0 = 0.3 m = aperture .

From reference [2] the SNR is given as a function of sampling rate and field diameter.
The centroid error is given in [1] as a function of the fwhm and the SNR of ref [2] by

sig_centroid = [4*ln(2)]^(-0.5) * fwhm / SNR

3.5 Off-Axis Guiding Error

Another measurement error is due to the fact that the guide star and science object are not both in the center of the field. Since the cassegrain rotator control system is imperfect, there will be a net RMS error in the cassegrain position. This rotational error of the field will make the guide object appear to move, and when the control system tracks the guide object, the error will be induced in the science object. Figure 6 shows the spectrum of the induced error in the image plane due to off-axis tracking.

3.6 Combined Error Spectra

The windshake, enclosure shake, and nonlinear control system induced shake are combined to give a total disturbance spectrum as shown in figure 7. This is for the worst case, TY30, and can be seen to be dominated by the windshake by comparing with figure 2.

Figure 8 shows the combined measurement noise spectra from figures 5 and 6, and can be seen to be dominated by the centroid error of figure 5.

4 RESULTS

There is a competition between disturbance errors and measurement errors. For a low sampling rate, and therefore a low system bandwidth, disturbance errors dominate the compensated RMS error. As sampling rate is increased, disturbance errors are reduced but measurement errors increase. For a given field diameter, there is an optimum sample rate which gives a minimum total RMS image plane error.

Figures 9 through 18 show the results versus sampling rate for the 10 different cases studied. Using figure 9 as an example, the two plots at the top of the page show the dominant error sources: windshake and centroid error vs. sampling rate. These two dominant effects are added, together with the lesser contributors described in section 3 above, to give the total RMS image plane error in the lower left of figure 9.

Table 1 summarizes the results for the 10 different wind cases, comparing the error budget with the minimum diameter necessary to meet the error budget (assuming that a sufficiently high sample rate were available). The cases labeled TY30 and TY45 are noticed to be very difficult, requiring about twice the available 200Hz sampling rate.

The first column of table 1 shows the wind case studied. The cases labeled TX# are crosswind cases with the telescope pointed # degrees above the horizon. The cases labeled TY# are looking into the wind and # degrees above the horizon. The second column of table 1 shows the error budget in microradians of RMS image centroid motion that is tolerable. The third column shows that diameter of guide field, in arcminutes, which theoretically could just barely meet the error budget at some sampling rate. The required sampling rate is sometimes quite high, as shown in column 5. For any given guide field diameter, there is a stellar visual magnitude such that there is a 90% probability of finding at least one star of that minimum brightness within the guide field. Column 4 shows the stellar visual magnitude based on the guide field diameter of column 3. The last column, column 6, shows that if the guide field were increased to 11.5 arcminutes the required sampling rates would be reduced. This is because a larger guide field would likely have a brighter guide star, which would reduce the amount of image smear due to the centroid calculation noise.

For the crosswind cases, labeled TX# in table 1, the required sampling rate increases with altitude angle because the error budget decreases near the zenith.

For the cases where the telescope is pointed into the wind, labeled TY# in table 1, the required sampling rate is higher for the lower altitude cases. Unlike the crosswind cases, the secondary support structure presents a larger surface to the wind at low altitude angles, which gathers more energy and causes more image plane motion.

Wind Case	Error Budget (microrads)	D_90 Min diameter to meet error-budget (arcminutes)	Stellar Mag required given D_90 (absolute)	Sample freq for D_90 to meet error-budget (Hz)	Sample freq for 11.5 arcmin to meet error-budget (Hz)
TX15	0.427	2.4	21.4	80	50
TX30	0.277	2.3	21.7	120	65
TX45	0.212	2.4	21.4	130	80
TX60	0.180	2.5	21.2	150	90
TX75	0.162	2.7	20.8	170	100
TX90	0.158	2.7	20.8	170	100
TY30	0.277	2.9	20.4	400	280
TY45	0.212	2.9	20.4	350	250
TY60	0.180	2.5	21.2	150	90
TY90	0.158	2.9	20.4	200	120

Table 1: Field Diameters and Star Brightness Needed to Meet Error Budget for Different Cases of Wind

The results of table 1 as well as figures 9 through 18 were obtained with the Matlab programs correl4.m and correl4b.m in the mburns directory c:\matlab\wind\.