To: Rick McGonegal

From: Mike Burns

Date: August 23,1993

Subject: Restriction Imposed on Tip-tilt for an Off-Axis Guide Star

Reference

[1] Private Correspondence between R. McGonegal and R. Racine August 20, 1993.

Introduction

If one attempts to do tip-tilt compensation on a science object by tracking a bright nearby object, what will be the effect of the angular difference between the two?

Summary

Unless an error has been made in calculating the results of this technical note, it is not practical to do off-axis atmospheric tip-tilt correction if 90% sky coverage is desired near the North Galactic Pole. It is assumed that % reduction in tip-tilt power is used as a measure of practicality. This may show that % reduction in tip-tilt power is not a good measure of improvement in the image. Starfire Lab's results show larger radii with significant Strehl improvement.

Calculations

There will be an error introduced in doing tip-tilt correction on one object but looking at a different nearby object. The requirement for the Gemini project is that the total atmospheric tip-tilt must be reduced in power to 10% of the uncompensated value.

From [1] the variance of error introduced by using an off-axis guide star is

sig_iso^2 = (x/r)^2 * sig_unc^2

where sig_unc is the uncompensated RMS, r is the correlation radius (arcminutes), and x is the angular difference (arcminutes) between guide star and science object.

The table below shows the normalized variance, that is by what factor the error power is attenuated when the difference between the guide star and science object is specified:

normalized distance (guide-science) = x/r	Diameter if isokinetic diam=3.4 arcmin	normalized tip-tilt power TTN= (x/r)^2	Effective tip-tilt pwr reduction TTPR	Req'd number of stars per square arcmin for 90% N	Required Visual Magnitude for 90% V
0.00	0.00	0.00	0.90	inf	inf
0.10	0.34	0.01	0.90	25.30	32.20
0.20	0.64	0.04	0.89	6.30	28.40
0.30	1.02	0.09	0.87	2.80	26.10
0.40	1.36	0.16	0.81	1.60	24.60
0.50	1.70	0.25	0.73	1.00	23.30
0.60	2.04	0.36	0.63	0.70	22.20
0.70	2.38	0.49	0.50	0.52	21.50
0.80	2.72	0.64	0.35	0.40	20.70
0.90	3.06	0.81	0.18	0.31	20.10
1.00	3.40	1.00	-0.01	0.25	19.50
inf	inf	n/a	n/a	0.00	-inf

To get a factor of 2 reduction in tip-tilt power requires a guide star within 0.7 correlation lengths, which in turn requires that stars to magnitude 21.5 be used.

In the above table, the effective tip-tilt power reduction represents how much of the noise is removed from the science object. If it is assumed that the guide star has 90% of its power removed, then the remaining noise is the root sum square of 0.1 and the normalized power of column 2. Thus the effective tip-tilt power reduction (TTPR) will be given by

TTPR = 1 - sqrt (0.1^2 + TTN^2).

Ideally, TTPR will be unity, representing all tip-tilt power removed from the science object. This is not possible though because of the assumed 0.1 residual power in the guide star, so the best practical TTPR=0.9. As the angular difference between the guide star and science object increases, the TTPR will shrink and eventually become negative, denoting that the process is actually adding noise to the science object.

The correlation radius (isokinetic radius) is empirically given by

$$r = \frac{0.3*Dtel}{hturb} = \frac{0.3*8}{5000} = 4.8e\text{-}4 \text{ rad} = 1.7 \text{ arcmin}$$

where Dtel = the telescope diameter and hturb is the turbulence height.

In order to have a 90% probability of finding a star within a given diameter D_{90} requires N stars per square arcminute.

$$D_{90} = 1.211/\mathrm{sqrt}(0.5*N) \quad \text{thus}$$

$$N = 2*(1.211/2x)^2$$

The number of stars per square arcminute with visual magnitude $<V$ at the North Galactic Pole is

$$\log_{10}(N) = 0.158*V - 3.68$$

$$V = \frac{3.68 + \log_{10}(N)}{0.158}$$

The above table has some interesting implications. At zero difference between the science object and the guide star, there is no noise introduced. This should be expected since it is assumed that the guide star is being tracked exactly. At the other end of the scale, things degrade badly. In fact, if the distance is great, noise is in fact amplified by 2. So when the science object is not near the guide star, we would do better to suffer with the noise as it is, rather than use a guide star. The mathematical break-even point seems to be at $x/r = 1.0$. It should be noted that the quadratic approximation to correlation is only for small off-axis distances. At $x/r=\text{infinity}$, one would expect the normalized tip-tilt power TTN=2 because science object and guide star will have no correlation. Clearly the quadratic approximation breaks down before $x/r=\mathrm{sqrt}(2)$ but is expected to be acceptable at unity.