GEMINI
8-M Telescopes
Project

TN-C-G0006

Chopping Secondary Control Study

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LIST OF FIGURES

SUMMARY

The chopping secondary servo system is required to provide 56.4 arcsecond (=270 micro-rad) motion at a rate of 10HZ. The required settling band of 0.485 micro-rad must be reached in 0.01 seconds to provide a usable duty-cycle of 80%.

A candidate controller was developed which meets the requirements, and the resulting model is used to estimate the average power. The following points are also noted: (1) digital control looks very troublesome at 200 Hz sampling; (2) 1kHz or more sampling is needed to try digital; and (3) continuous control needs a 640Hz bandwidth estimator and 75 W net power.

Summary of Power and Duty-cycle for Various Controllers

Controller type	Power- 1 actuator (W)	Net Power (W)	Duty-Cycle
Original PID-Benign	5	8	80
Original PID-parasitics	20	30	72
PID-increased gain	32	48	79
Digital PID	3	4	0
Kalman filter/Optimal ctrl	50	75	88

SYSTEM MODEL

The basic model of the servo-chopper system is nearly the same as that described in Chopper Power Requirements (Appendix 1) with the addition of some parasitic terms. The calculations of torque and power also have been changed slightly.

Figure 1 shows the generalized top-level view of the simulation used for this study. Input is modeled as a 270 micro-rad step that is subtracted from the actual angular position of the secondary to produce an error, labeled Theta-eff. The controller block, expanded in **Figure 2,** acts upon the error to produce a limited command voltage, Vlim, which feeds the servo block of **Figure 3.** The servo block produces Torque delivered to the Chopper of **Figure 4** and an intennediate variable, Ilim, the limited current used by the Power block of **Figure 5.**

The overall simulation is "mixed" in the sense that part of the simulation describes a single actuator and part of the simulation describes the summed effect of three actuators. The Controller, Servo, and Power blocks model one actuator, while the Chopper block models the effects of all three actuators. The crossover from part to whole occurs in the last gain block of the servo, Figure 3, which is labeled i2trq. The gain i2trq is the scale factor from current in one actuator to net torque produced by three actuators.

Figure 6 helps to describe the relationship i2trq. Assume that the three actuators are placed in the geometry shown and produce forces labeled Fl, F2, and F3 Newtons. The desired rotational axis is labeled xx' and the moment arm is z=0.3 meters. The fact that torque is about the xx' axis requires that:

F2 = F3.

Net force on the mirror must balance, requiring that

Fl + F2 + F3 = 0.

Combining the above 2 equations gives

F2 = F3 = -F1/ 2.

Noting that sin (30 deg) = 0.5, the diagram makes it clear that net torque will be

Tnet = F1 *z - F2*z/2 - F3 *z/2 or

Tnet = Fl*z + Fl*z/4 + Fl*z/4

Tnet = F1 *z*3/2.

For this possible actuator, the relationship between current and force is 35N/A, so the previous equation can be written

Tnet = I1 * 35N/A * 0.3m * 3/2 = I1 * 15.8,

showing that the gain i2trq = 15.8N*m/A. This is half the value used (see Appendix 1), but is believed to be more accurate. By way of an argument similar to the one above, the net power may be shown to be 1.5 times the power in the actuator number 1.

Note that an alternate geometry would be to draw the axis of rotation through one of the actuators in Figure 6. It is not shown here, but the resulting i2trq = 18.2. The 15.8 value is chosen because it is the more pessimistic (hitting the current-limit and torque-limit sooner). In general, the geometry would influence controller gains between these extremes.

The Servo block of Figure 3 also shows two other small changes (see Appendix 1): (1) addition of a back-EMF term, and (2) an electrical time constant, taue = 0.25 milliseconds.

A first guess at the controller model is to use the PID controller (see Appendix 1). Note that gains appear doubled because this controller goes from theta-eff to volts, whereas the reference went from theta-eff to current. Current and voltage are related by the 2 ohm armature resistance. The derivative term of the controller is changed from a pure differentiator to one with a very small time-constant, taud = .3ms to reflect the lag inherent in measuring or calculating speed. The controller also includes a voltage limiter which was not modeled in the reference, but is always part of a power amplifier which will drive the servo. Note that the current limit is also modeled, but is not sufficient. Roughly, the current limit restricts net force and voltage limit restricts rate of change of force.

The chopper of Figure 4 is fundamentally unchanged from the stated Chopper Power Requirements (Appendix 1), and the Power block integrates I*I*R and divides by the 0.05sec half-wave time to get average power.

To sumirnarize, the basic model used in this study is the same as that of the Chopper Power Requirements (Appendix 1), except for the following changes:

- i2trq changed from 31.5 to 15.8 N/A
- voltage limiting
- back EMF in servo
- servo dynamic time constant of taue = 0.25 ms
- controller differentiator time constant, taud = 0.3 msec.

SIMULATION RESULTS AND THE SEARCH FOR A GOOD CONTROLLER

Original PID-Benign case

Figure 7 shows simulation results for the most benign possible case, omitting voltage limiting, back EMF, servo electrical time-constant, and having negligible (0.03 ms) differentiator time-constant. The measured outputs are from top to bottom: Theta-error (rad), servo voltage (V), servo current (A), mirror angle (rad), angular rate (rad/sec), and average power (W). **Figure 8** redraws the error on a better scale and shows that the spec is met since error falls below 0.485 micro-rad before 0.01 sec. Despite the integral term, the MatrixX implementation of this PID controller has a non-zero steady state error. This error is likely due to quantization effects within MatrixX or to the very tiny integral gain which causes an error to be reduced very slowly. The resulting power is 5.1 W.

Original PID-with parasitics

Figures 9 through 18 show the effects of gradually adding more of the parasitic effects. The voltage limit and back-EMF have little effect on power - perhaps reducing power marginally - and make the system slightly slower so that the 0.485 micro-rad settling band is not reached until 0.012 sec. The addition of an electrical time-constant has increases power consumption 40% to 7W. The most devastating effect upon power comes from the controller differentiator lag, taud. Increasing it to 0.1 ms then to 0.3 ms increases power to 9.5W and 20W. If taud is increased to 1ms, the controller is hopeless, with power increasing to 65 W and settling time going to 0.04 sec.

Taud = 0.3 ms was chosen rather arbitrarily as a good representative value of a realistic sensor without greatly harming performance.

PID-increased gain

Can the structure of this controller be saved and adequate perfon-nance restored by changing gains? The root locus plots of **Figures 19-24** help in describing this system. The first three of these figures show the root locations of the benign system for changing the proportional, integral, and derivative gains by the scale factors shown on the plots. It is interesting to note that the integral gain may be changed by a factor of 20 while moving the root location negligibly. This indicates that the original controller was nearly a PD type. Since the derivative time constant, taud, was the most important of the parasitic elements modeled, its root locus is shown in **Figure 22.** Predictably, things get much slower for taud = 0.3 ms than for 0.03 ms. **Figures 23 and 24** show the proportional and derivative root loci again, this time for the less benign case of taud = 0.3 ms. Increasing either the P or D gain improves the root on the real axis near -400 at the expense of the lightly damped root near 400 +j1600. Increasing the P gain by a factor of 1.5 seems to give the best tradeoff, with time responses shown in **Figures 25 and 26.** The settling time has been restored to near 0.0105 sec, but power has increased to near 32 W.

Digital control

Thus far only a continuous time controller has been modeled, but in the actual system there are strong reasons for using a digital controller, notably ease in changing parameters and stability of coefficients. **Figure 27** shows the root locus of the discretized system with a 200Hz sampling rate and an extra step delay to represent computational lag. **Figure 27** was found to be poor because of the root near -0.8 at the left of the plot tending to go unstable as gain increased. **Figure 28** shows the effect of adding a lead-lag compensator to pull in the errant pole. Still, the system response of **Figure 29** is poor. Note that at 200Hz the controller has precisely two steps in which to reduce the system error by a factor of 0.0018 (from 270 to 0.485 micro-rad), so the effective system root must be near sqrt(0.0018) = 0.04. This is so close to deadbeat as to be unattainable with a linear controller, so a bang-bang type controller was examined for the first two steps handing off to a slower linear controller to maintain good noise response.

Figure 30 shows the MatrixX block diagram of a controller which was cascaded with the linear digital controller to implement the bang-bang function over the first two time steps. The bang-bang control used here is slightly more sophisticated than that described in the reference because chopper is modeled as inertia-spring-damper rather than merely inertia. A few words about derivation of the control steps are given below.

The chopper may be represented by the discrete state space equation

x(k+ 1) = PHI * x(k) + gam * u(k)

where x(k) is the state [theta(rad) theta-dot(rad/sec)]' and u(k) is the applied torque. It is desired to find torques u(0) and u(1) such that the system is moved from its original state x=[0 0]' to the state [2.7e-4 0]' . The state space equation may be iterated to give

x(l) = PHI * x(0) + gam * u(0)
x(2) = PHI * PHI * x(0) + PHI * gam * u(0) + gam * u(1).

The last equation above is a simple linear equation in the unknowns u(0) and u(1) and may be rearranged and solved:

x(2) = PHI * PHI * x(0) + [PHI * gam | gam] * [u(0) u(1)] ' or
inv[PHI * gam | gam] * [x(2) - PHI*PHI*x(0)] = [u(0) u(1)].

Solving the above gives the necessary step torques of u(0)=43.618 Nm and u(1)=-41.321Nm. This is quite close to the simpler values derived in the reference of +/- 43.68 Nm, showing that inertia is by far the dominant term. Neglecting the effect of the electrical time constant, the computed torques correspond to commanded voltages of 5.521V and -5.231V respectively. Unfortunately, when these are applied to the system the effect is completely unsatisfactory as shown in **Figure 31.** The large transient response in theta-eff is due to the neglected state associated with the electrical time constant. **Figure 32** verifies that the system response is excellent, with theta-err=0.03 micro-rad, when the electrical lag is removed. The state equations could be rewritten include the effect of the electrical lag, but it would require at least three time steps to move all three states to the arbitrary location, violating the 0.01 sec required settling time.

In general an N-state system will require N steps to move from one state to another specified state. Even if we were to build a controller that would take into account the known states, umnodelled states would remain which would cause hw-m analogous to that shown above. A fast linear controller would still be needed to take out the system error. For example, with a 1kHz sampling rate, it would take 10 steps in which to remove error. The first three steps could be used to take out the three known states, and the other seven steps could remove the remaining error due to unmodeled effects. Due to actuator current limit - and thus torque limit - three steps might be insufficient time in which to move the system. In such a case more steps could be used, or the effective torque limit could be increased by including four or more actuators in a different geometry.

A 1kHz digital filter should easily be implemented on the current system. For an N-state filter, there are required roughly n^2+3n floating point multiplies and as many floating point adds. So a five-state filter would need approximately 40 multiplies and 40 adds. Assuming a 68030 processor operating at 25MHz, there are 25,000 clock cycles in a 1ms sample time. Conservatively estimating that a floating point multiply takes 64 clock cycles and an add takes 44 clock cycles, a total of 4,000 clock cycles for the filter is the result. Doubling this to 8,000 clock cycles to conservatively estimate the necessary software overhead still uses only 1/3 of the available CPU power.

Kalman Filtering/Optimal Control

Since the above analysis shows that a 200Hz digital system has little hope of working, a Kalman-filtering/optimal-control type approach was considered in the continuous domain. **Figure 33** shows the resulting system with the estimated states being chopper-angle, angle-rate, and servo-current. Properly speaking, the system shown is neither Kalman-filtering nor optimal-control because the estimator gains ke and controller gains kc are not chosen based on measurement or system noise. Still, the structure is the same. **Figures 34 and 35** show the time responses associated with the first iteration of this approach, with controller gains which give poles at -1000 and -700 +/-j714 and estimator gains which give poles at -2000 and -1400 +/-j1428. The transient response is seen to die out quickly enough in six ms, but there is left a steady-state error of 1 micro-rad, well above the specified 0.485 micro-rad. One alternative is to add another state, integrated error, and redo the filter and controller gains for the resulting four-state system. A more simple approach was taken in just making the controller stronger. **Figures 36 and 37** show the effect of moving the controller poles to -2000 and -1400 +/-j1428 and the estimator poles to -4000 and -2800 +/-j2828. Now the steady-state error is only 0.25 micro-rad. If this steady-state error were unacceptable, you could cascade the limited PI controller of **Figure 38.** The limiter before the integrator insures that it will not become highly charged by the initial violent transient, and will only affect the long term response as shown in **Figures 39 and 40.** The estimator pole at -4000 rad/sec is equivalent to a bandwidth of 640Hz, which should easily be implemented.

Notes on Scaling

All of the controllers presented here are likely to have a much easier time at a slower chopping rate. Since every controller has an initial transient response similar to bang-bang, it seems reasonable to consider the equation from the reference :

$$T^2 = 4 * theta * J / torq$$

where T is the travel time, theta the angle to be traversed, J the moment of inertia, and torq the maximum torq. Assuming that most of the power is used in the initial transient and letting "~" represent "is proportional to" the following dependencies are noted:

$$Pav \sim Pmax \sim current^2 \sim torq^2 \sim I / T^4 \sim theta^2.$$

For example, with a 5Hz chopping rate and a 540 micro-rad angle, the net effect is that power is reduced by a factor of four. Note that this assumes that the torq limit (current limit) is similarly reduced for the slower chopping case. For those controllers which meet the required settling time, this would be allowable. The response could be accelerated by allowing a comparatively larger torque limit at the expense of more power, offering hope for some of the slower controllers which barely missed meeting the settling time.

Future Work Possibilities

This study addresses the basic issues of chopping control. There are a number of areas that would benefit from additional work:

- examine plant and measurement noise effect on Kalman filter approach
- model power amplifier dynamics, possibly including a state in the estimator
- find effect of reaction force (torque doublet) on structure
- find tracking bandwidth and residual error spectra of resulting closed loop system
- look at the effect of Coulomb friction upon tracking accuracy and power
- test robustness to modeling errors such as chopper spring constant
- consider a digital controller at higher sampling rate and a 5-state estimator to model computational lag, amplifier dynamics, and electrical lag

Figure 1. General Top-level Simulation View

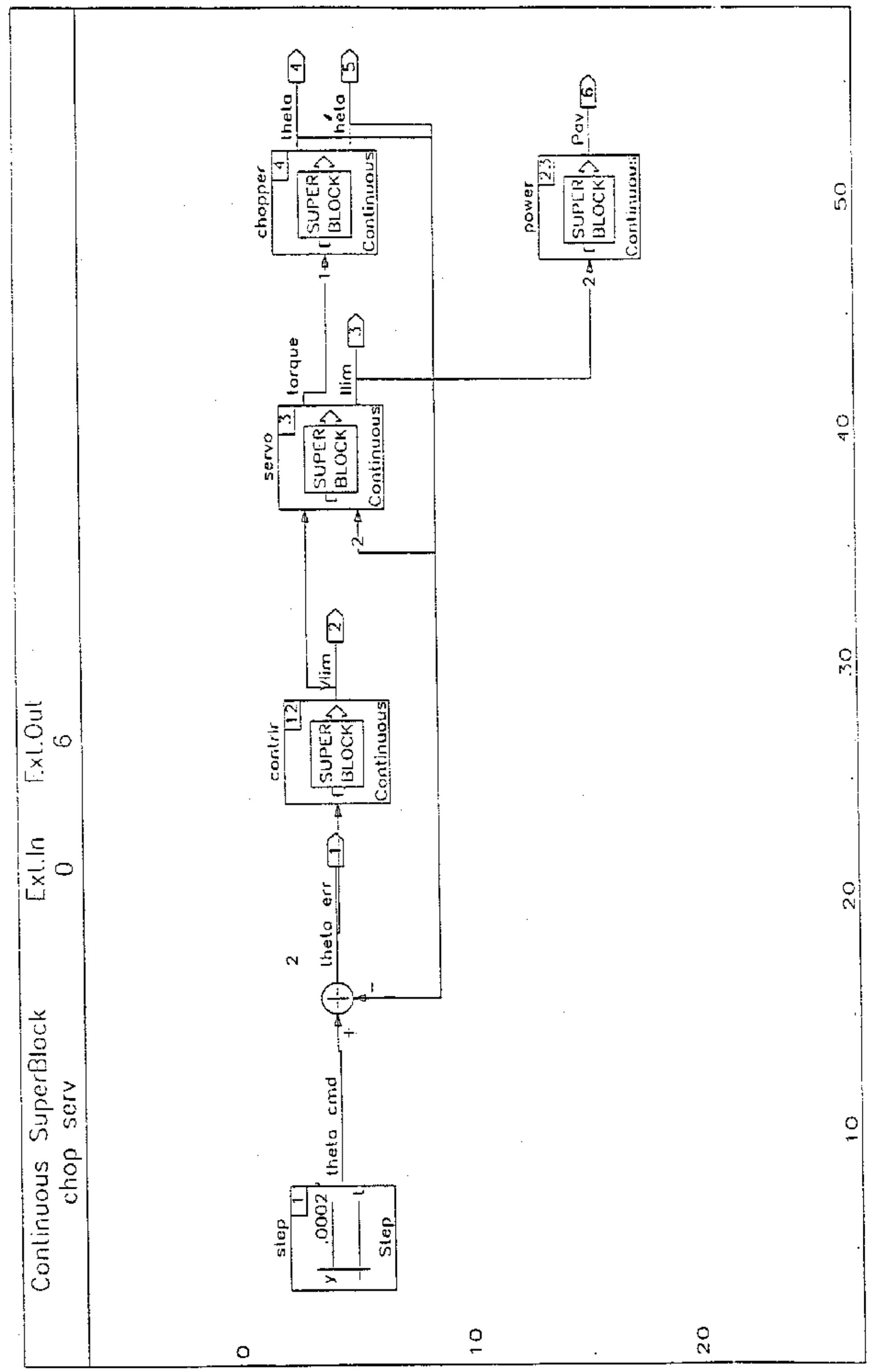


Figure 2. Expanded Controller Block

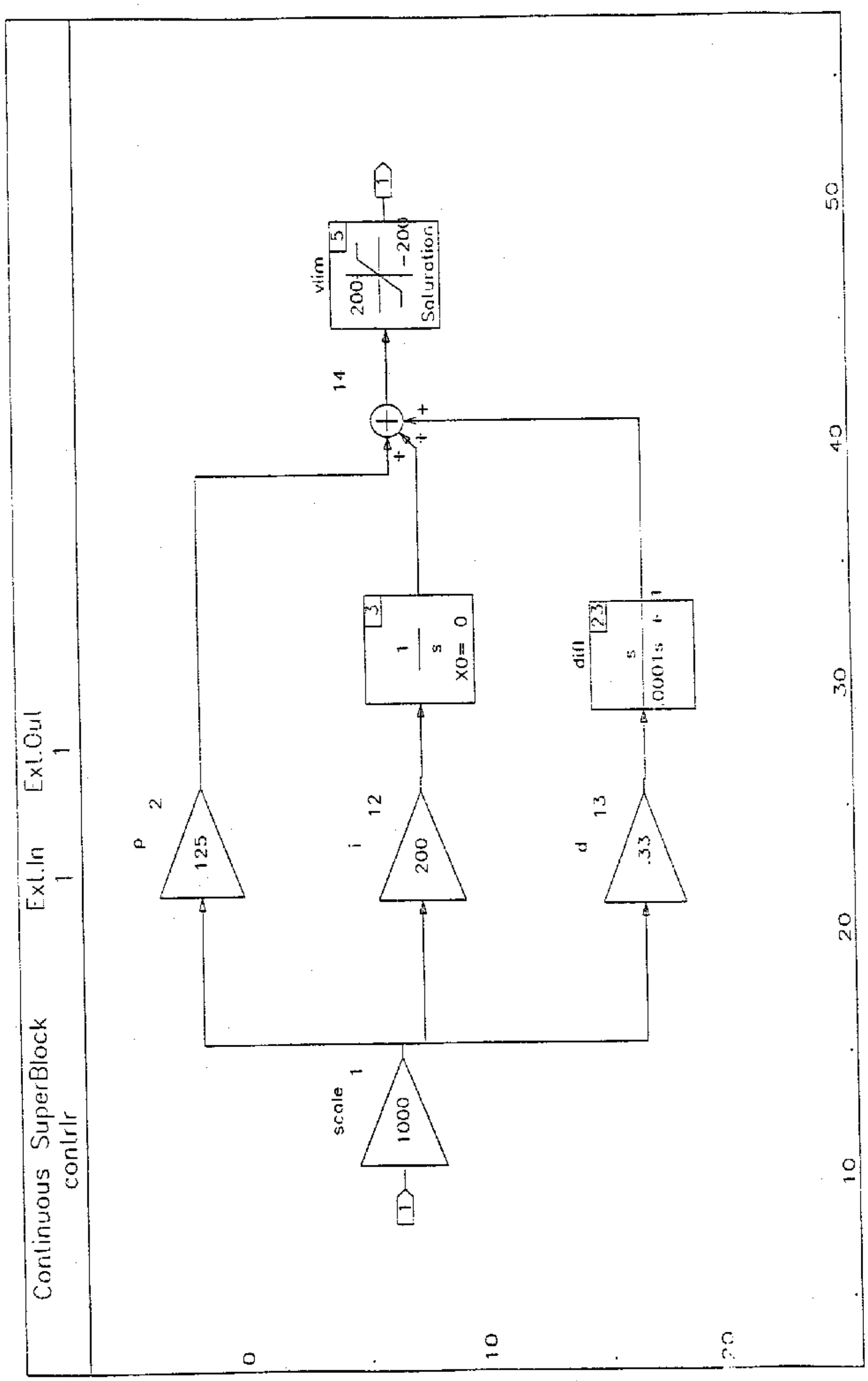


Figure 3. Servo Block.

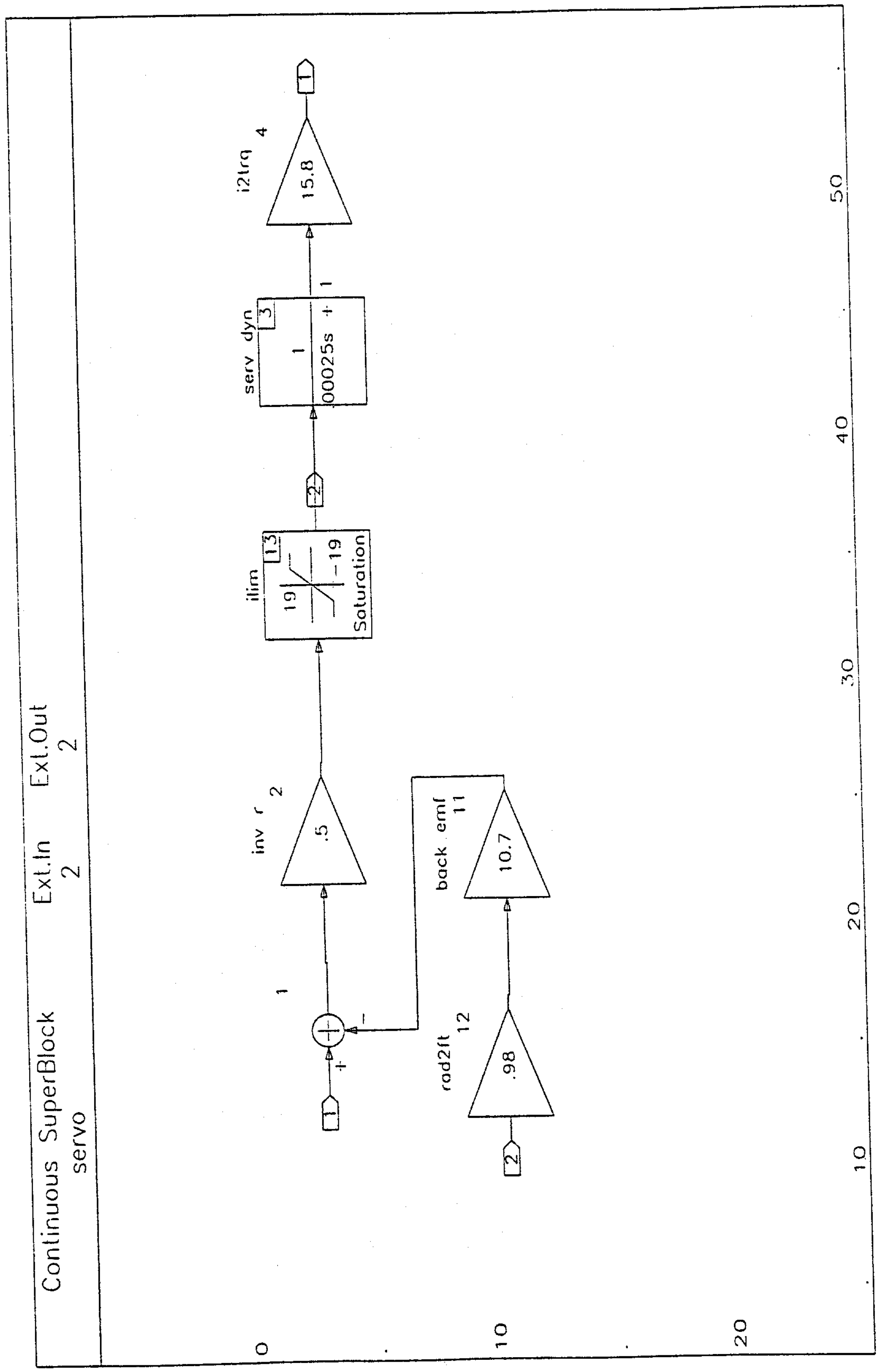


Figure 4. Chopper Block

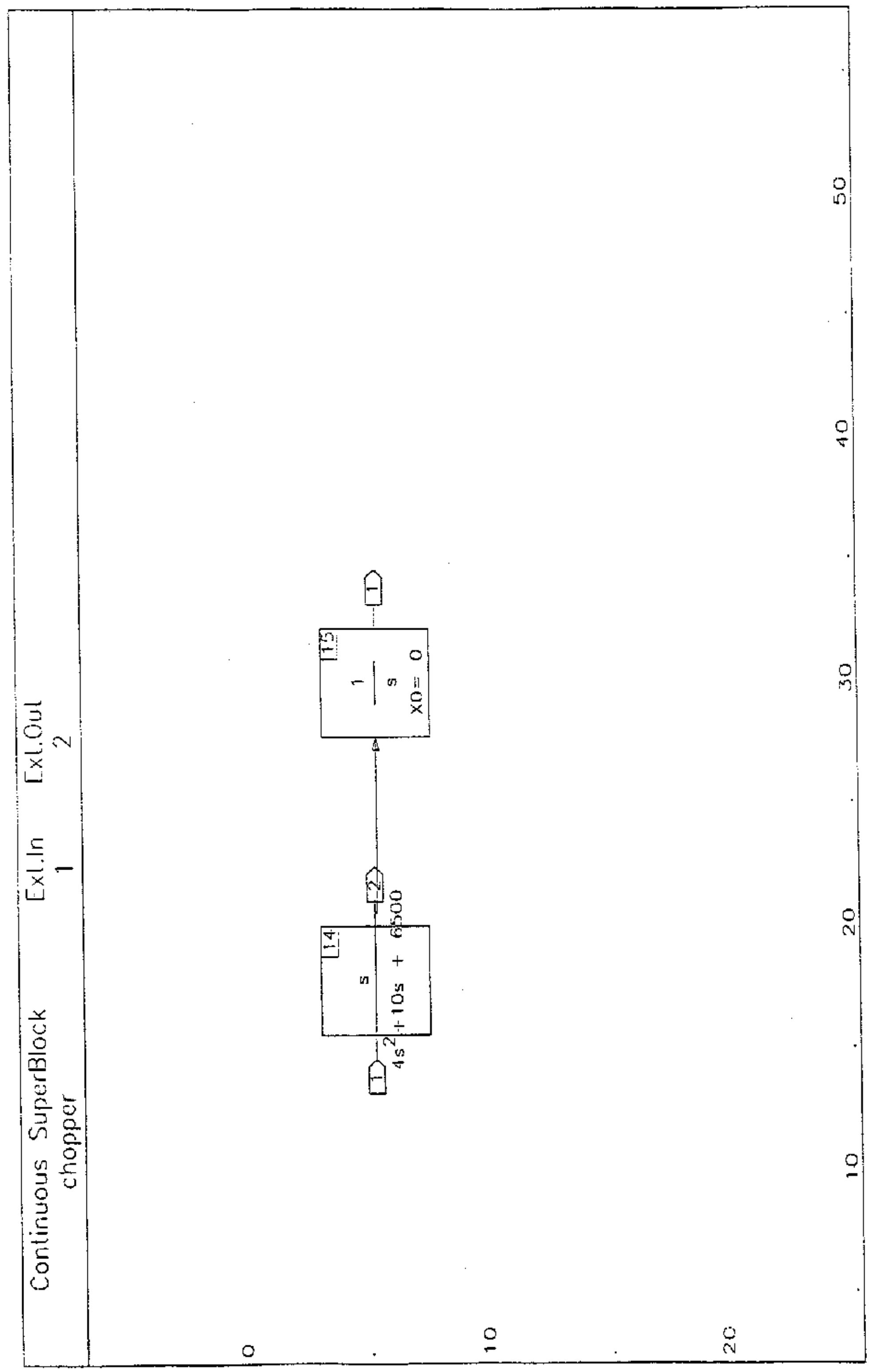


Figure 5. Power Block

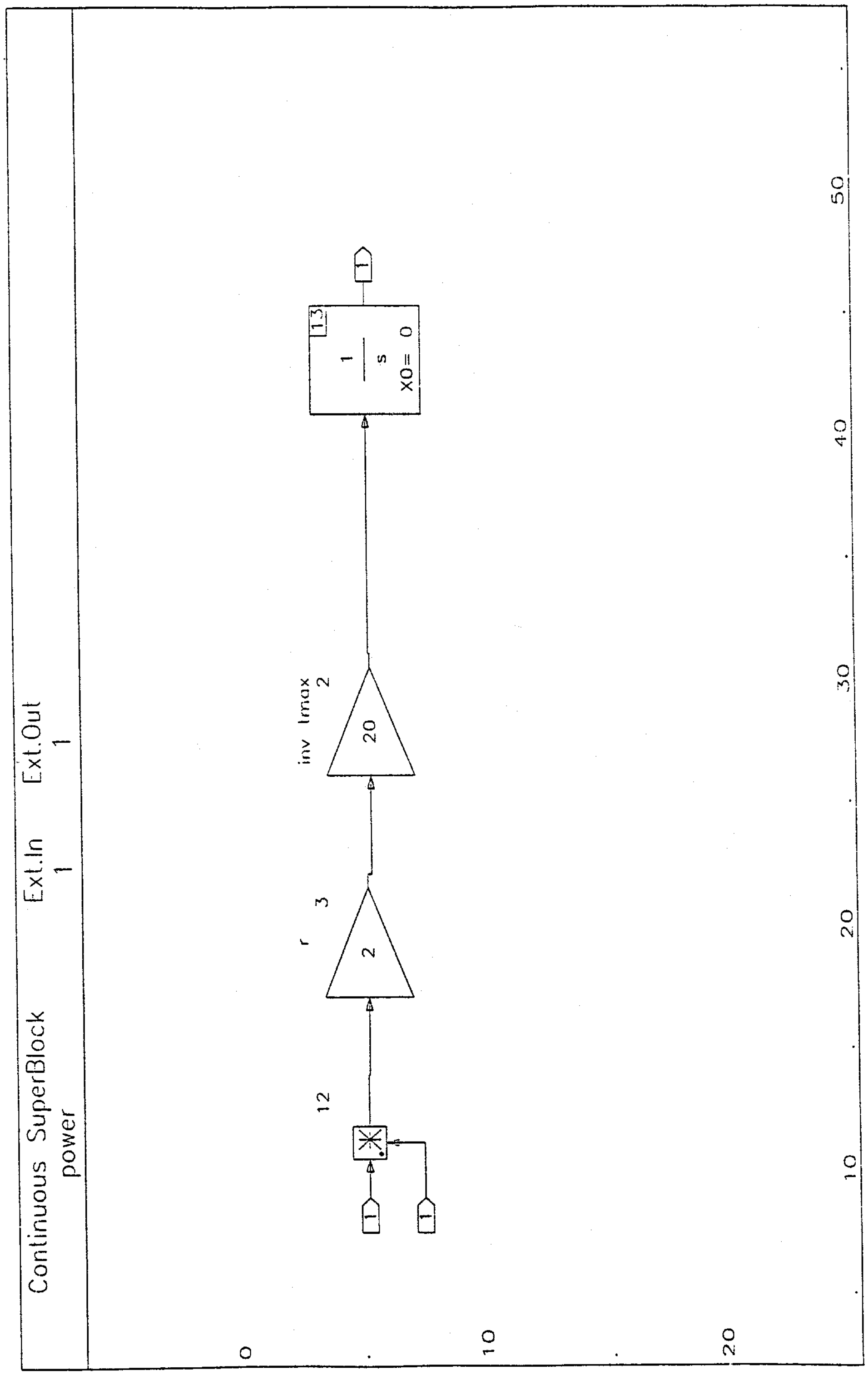


Figure 6. Net Torque Calculation

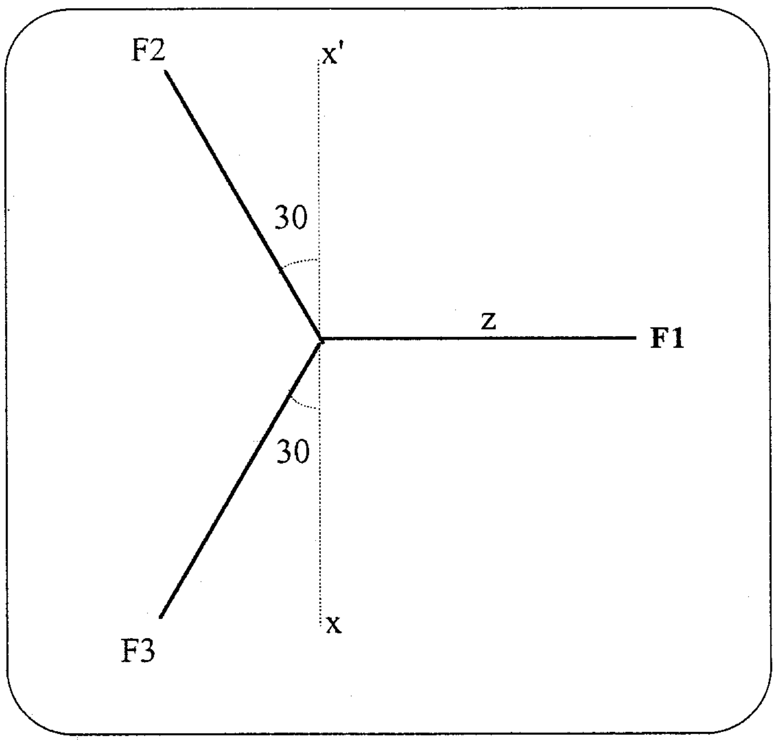


Figure 7. Benign Case Simulation Results

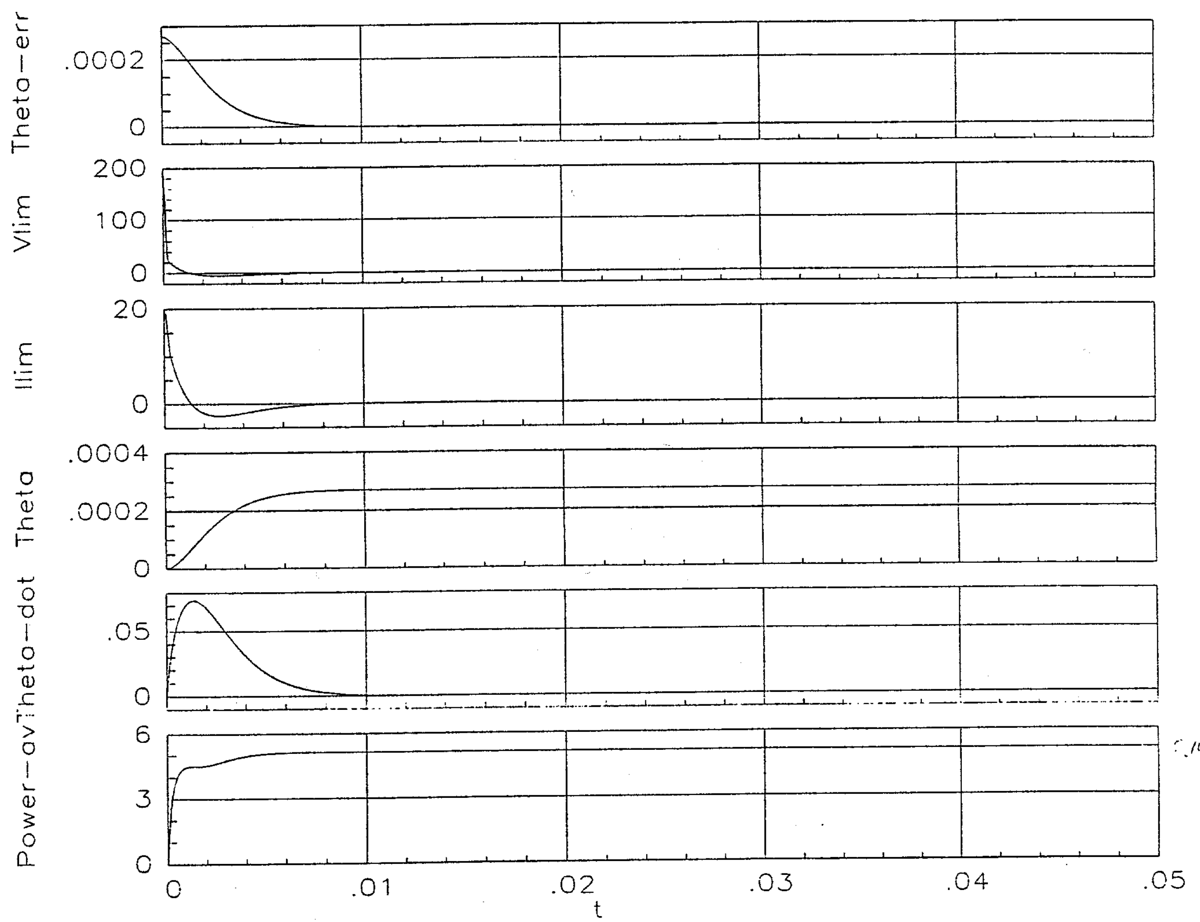


Figure 8. Error vs. Time for Benign Case

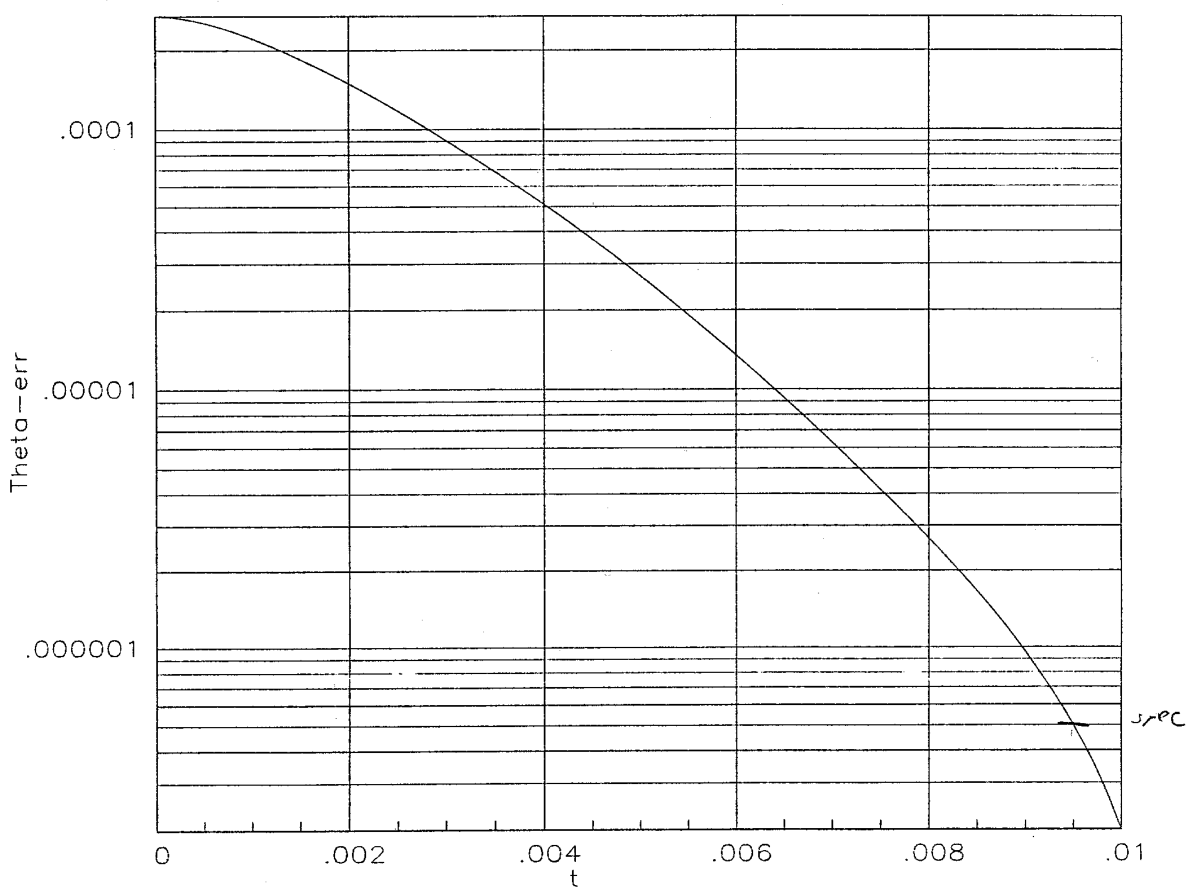


Figure 9. Benign + Vlim + VB Simulation Results

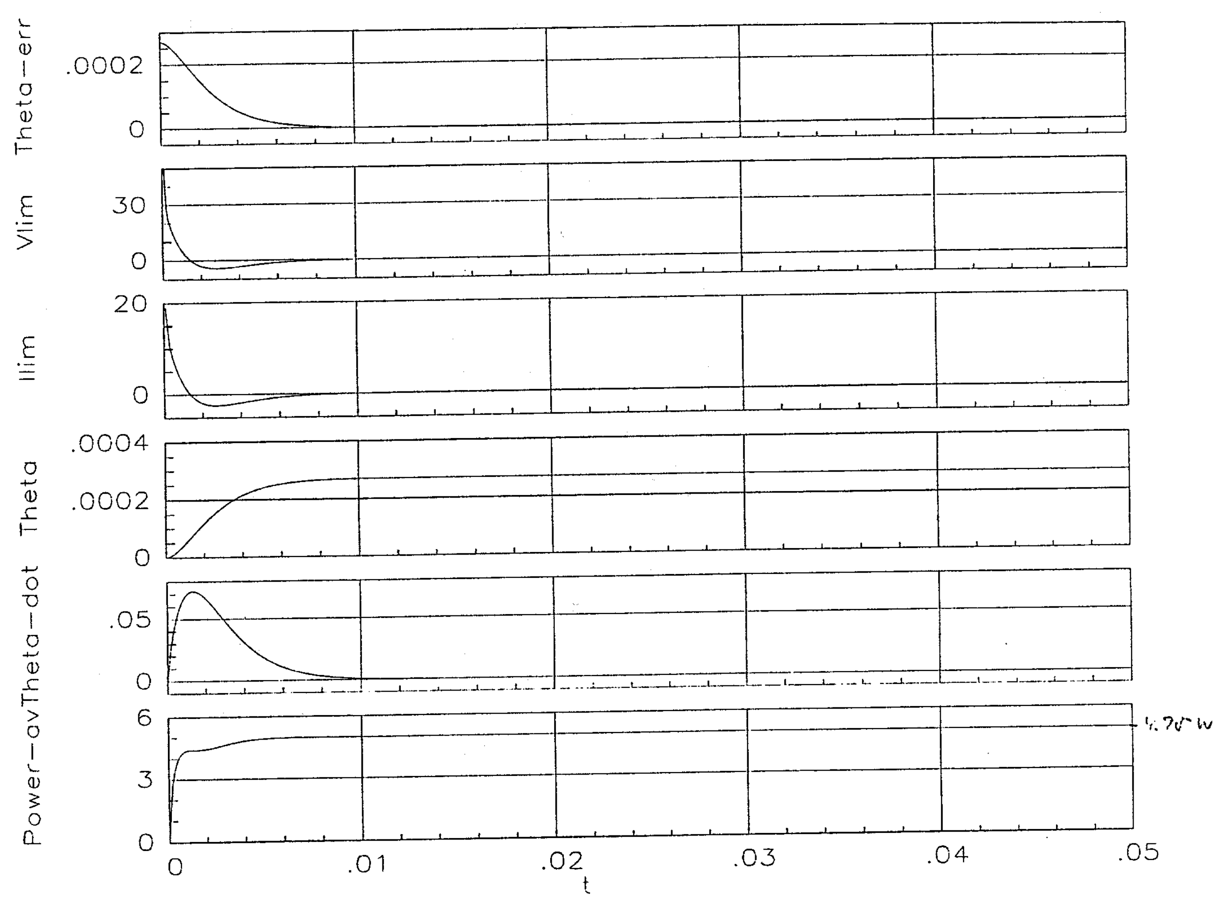


Figure 10. Errorvs. Time for Benign + Vlim + VB Case

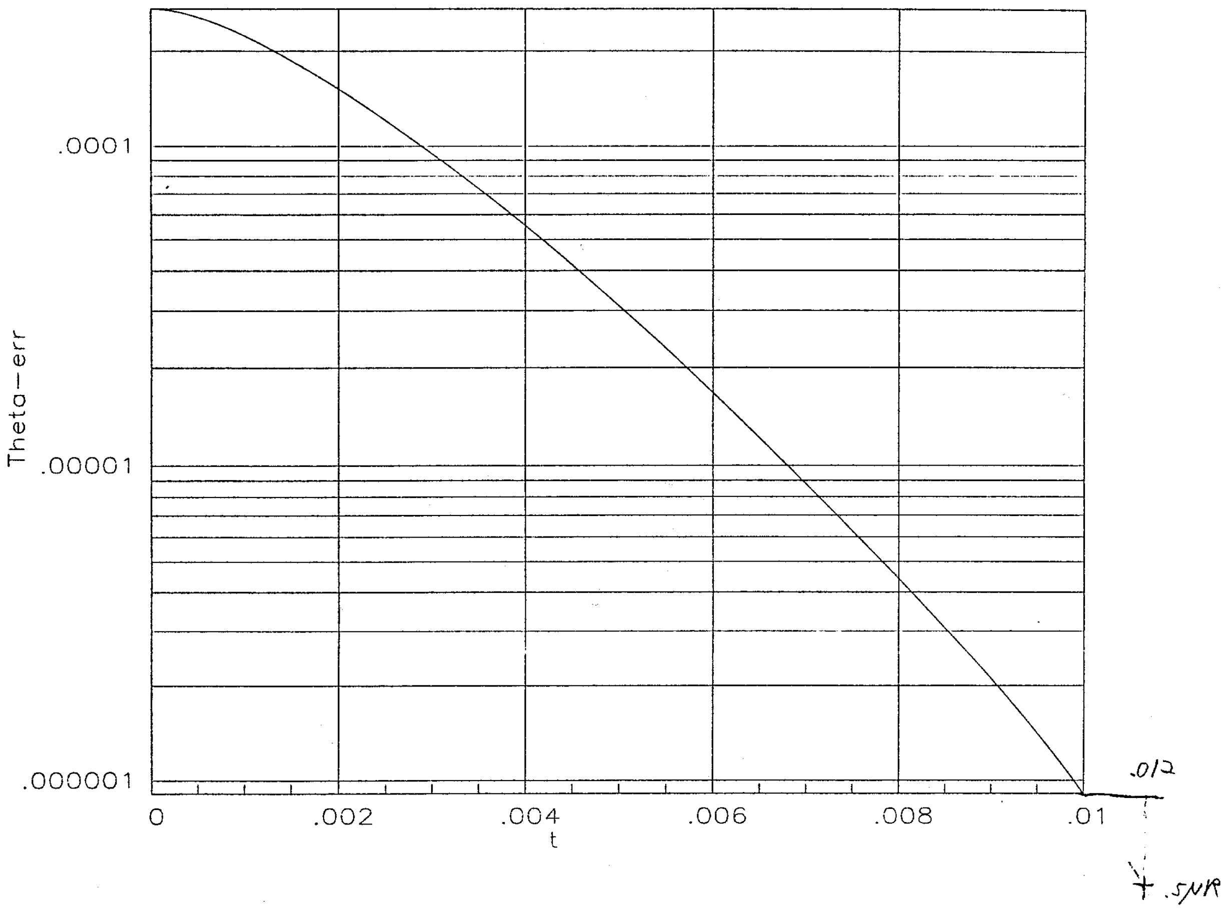


Figure 11. Benign + Vlim + TE Simulation Results

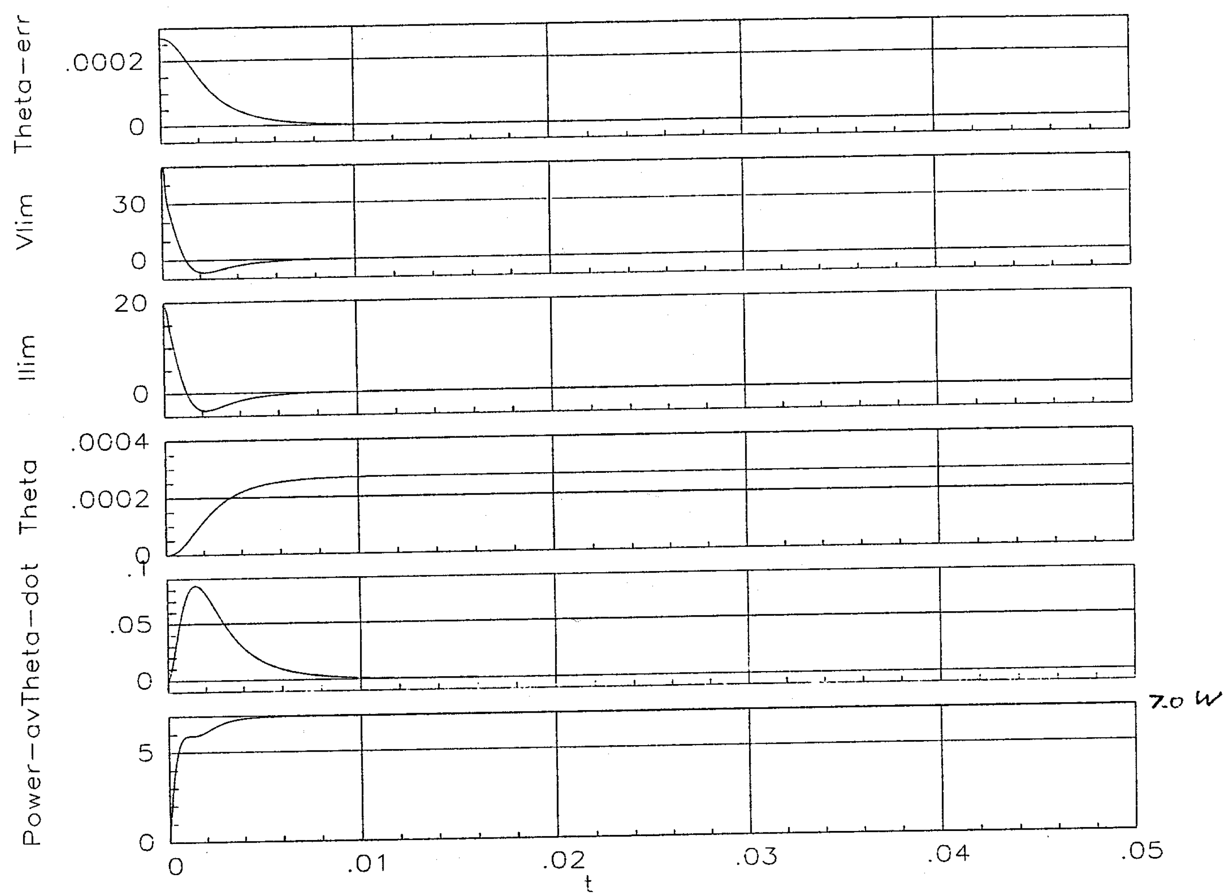


Figure 12. Error vs. Time for Benign Vlim + VB + TE

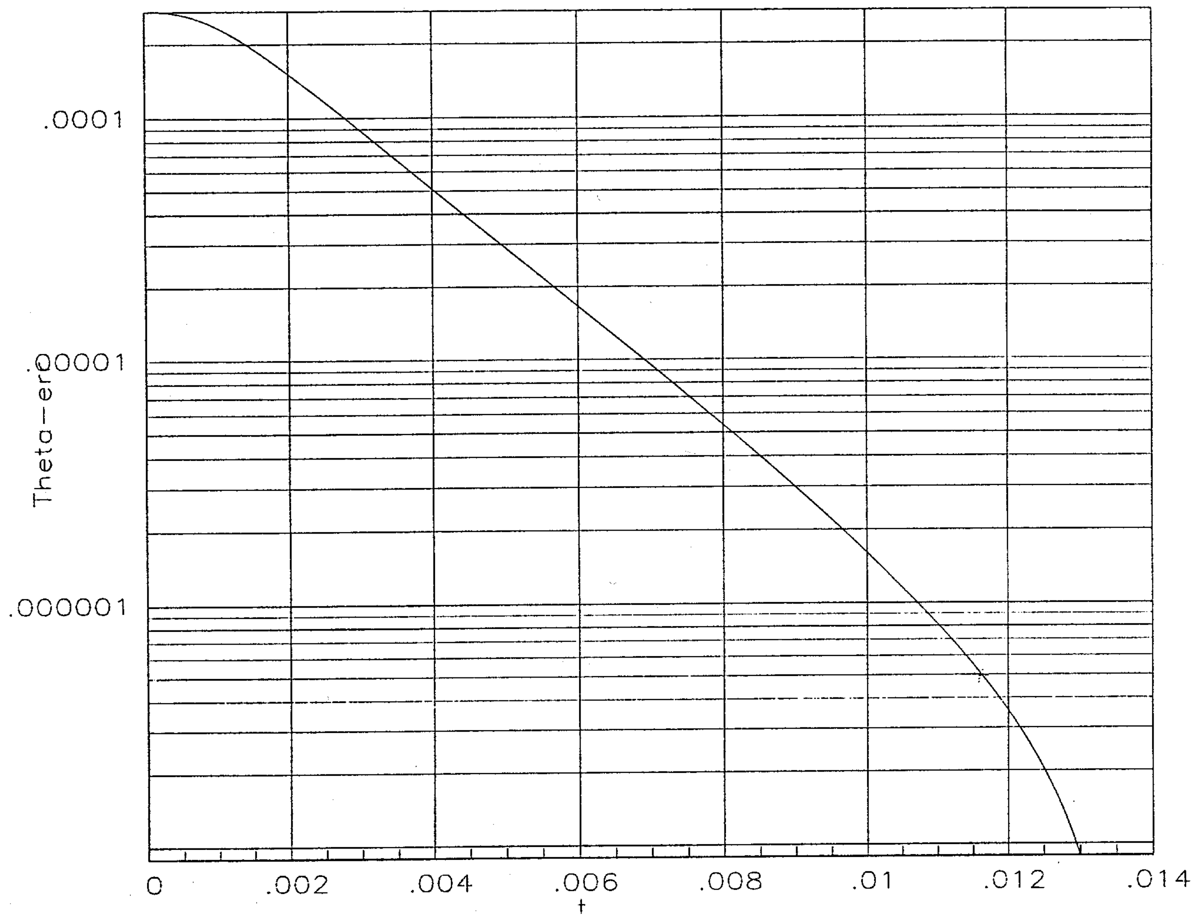


Figure 13. TD=0.1ms Simulation Results

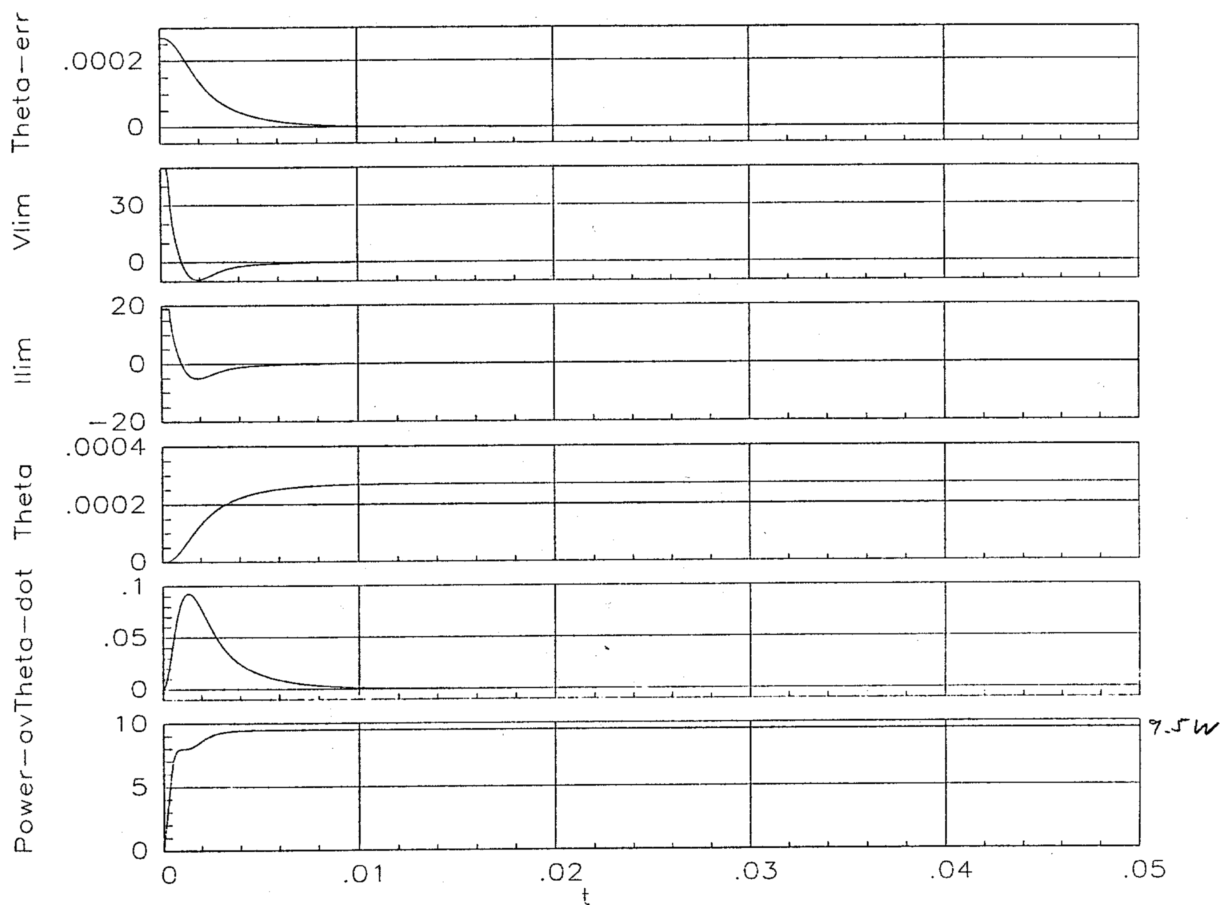


Figure 14. TD=0.1ms Error vs. Time

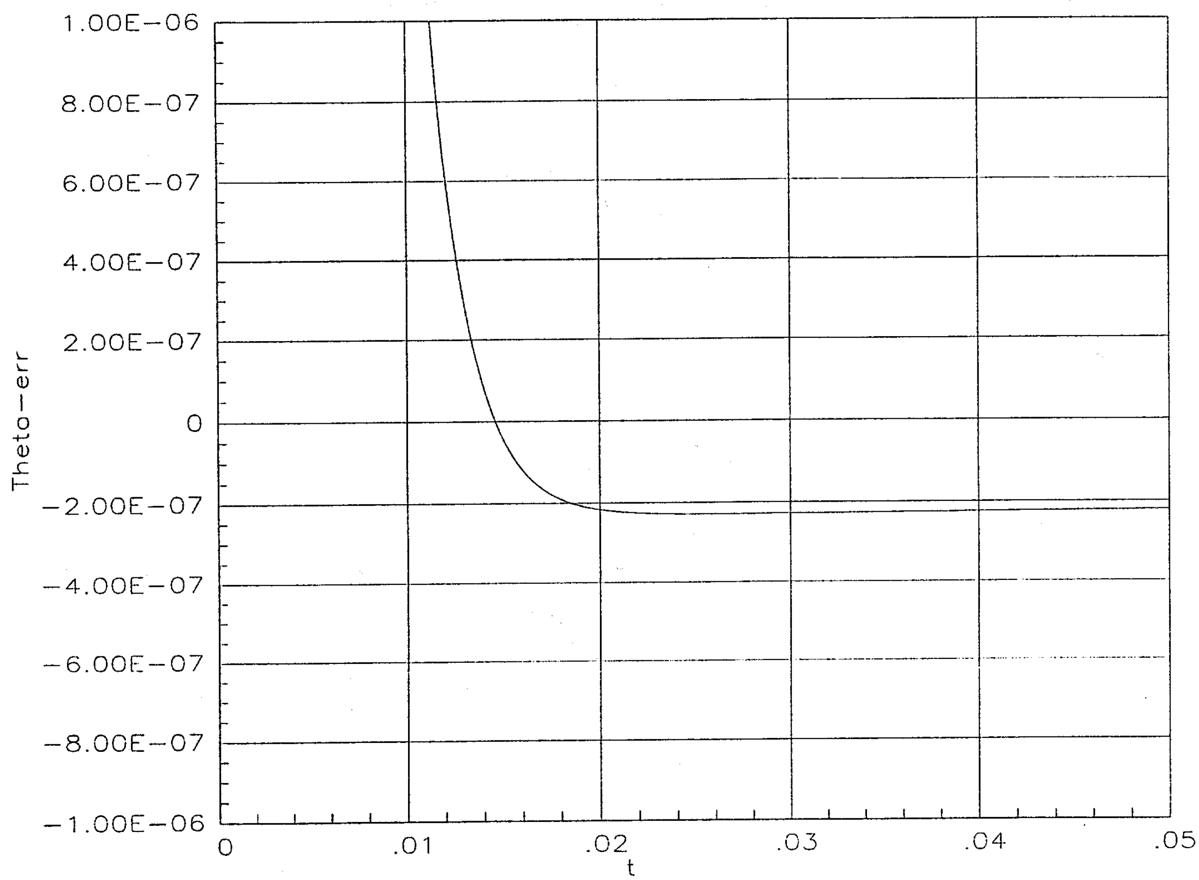


Figure 15. TD=0.3ms Simulation Results

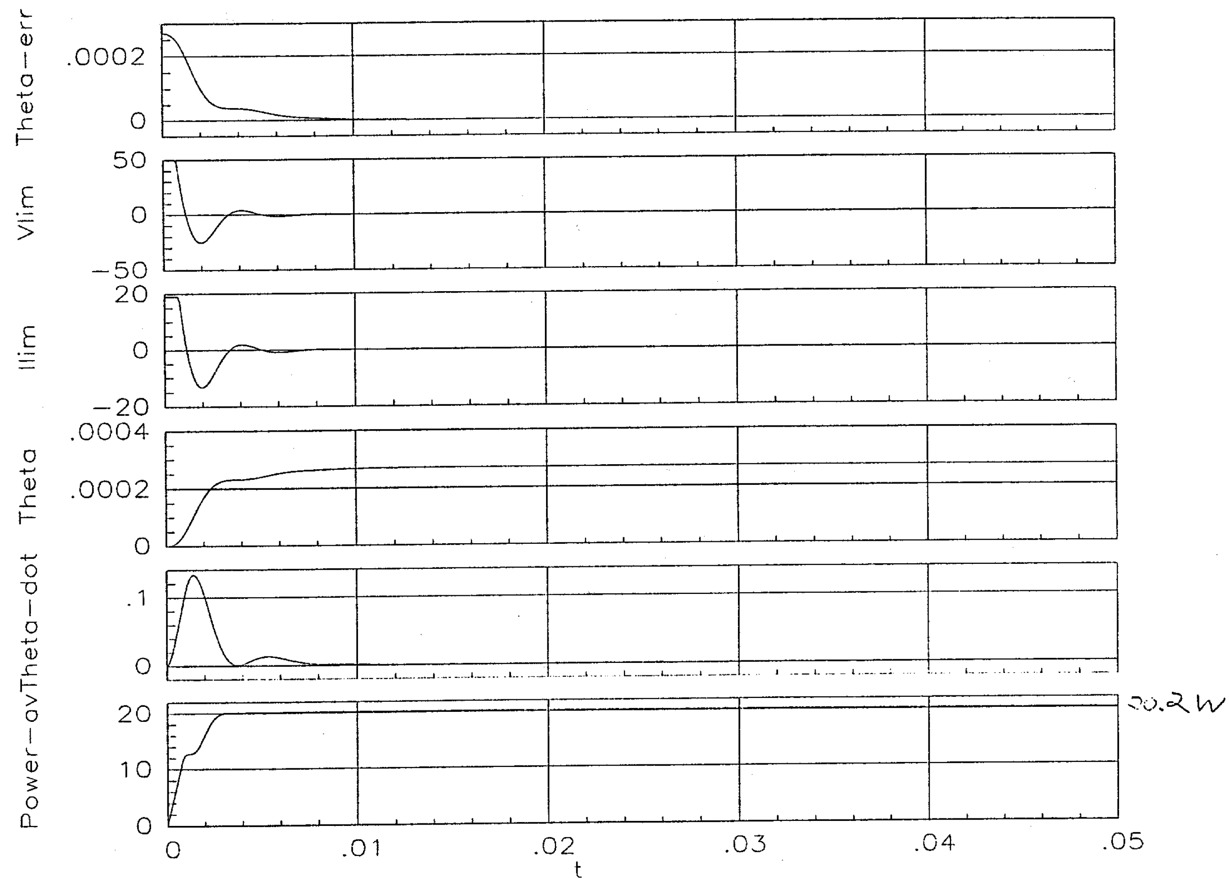


Figure 16. TD=0.3ms Error vs. Time

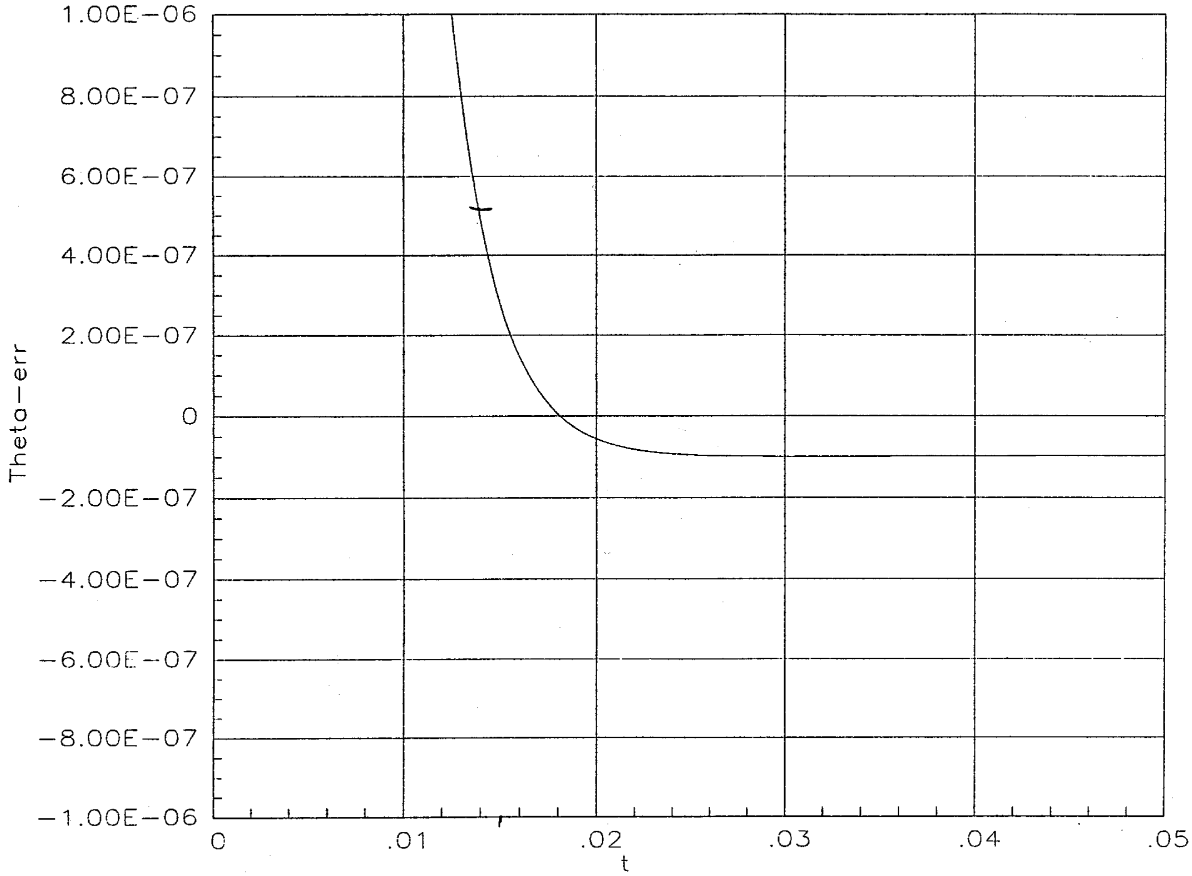


Figure 17. TD=1.0ms Simulation Results

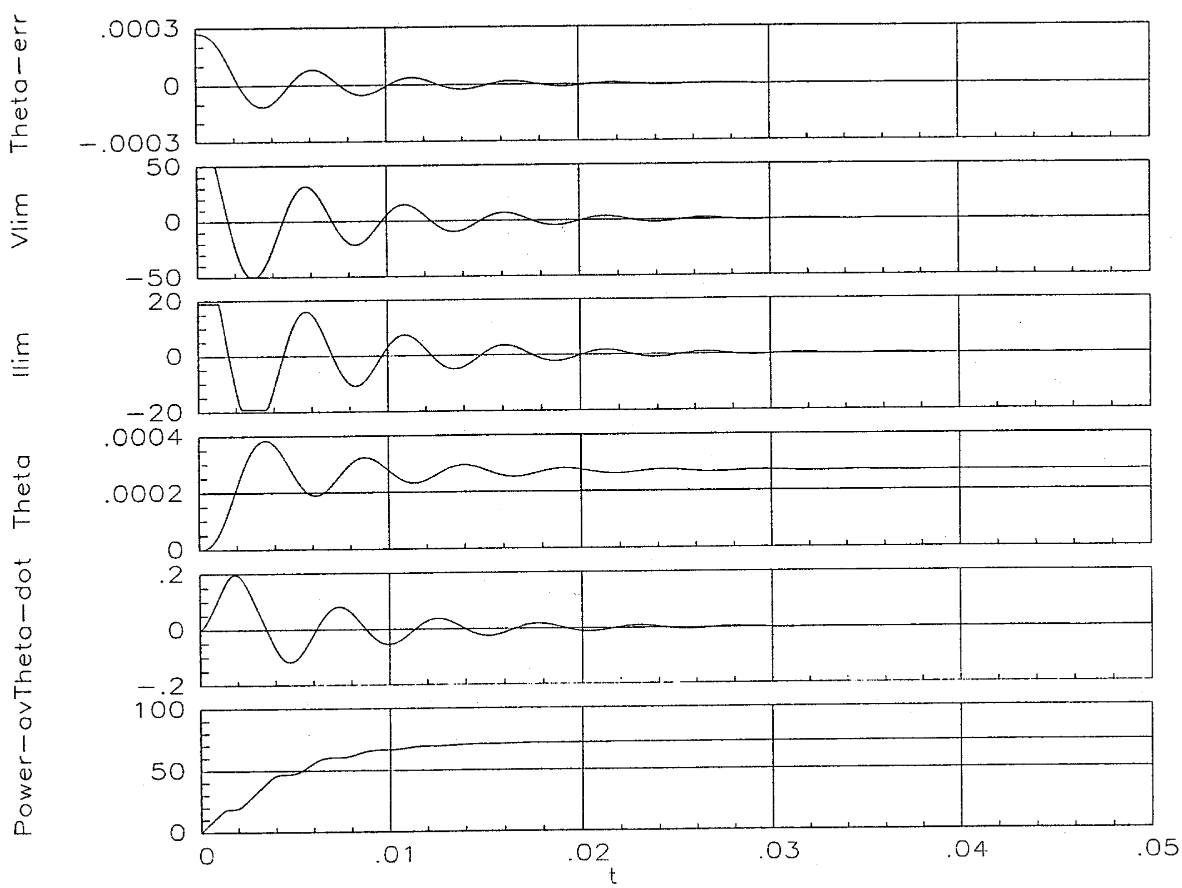


Figure 18. TD=1.0ms Error vs. Time

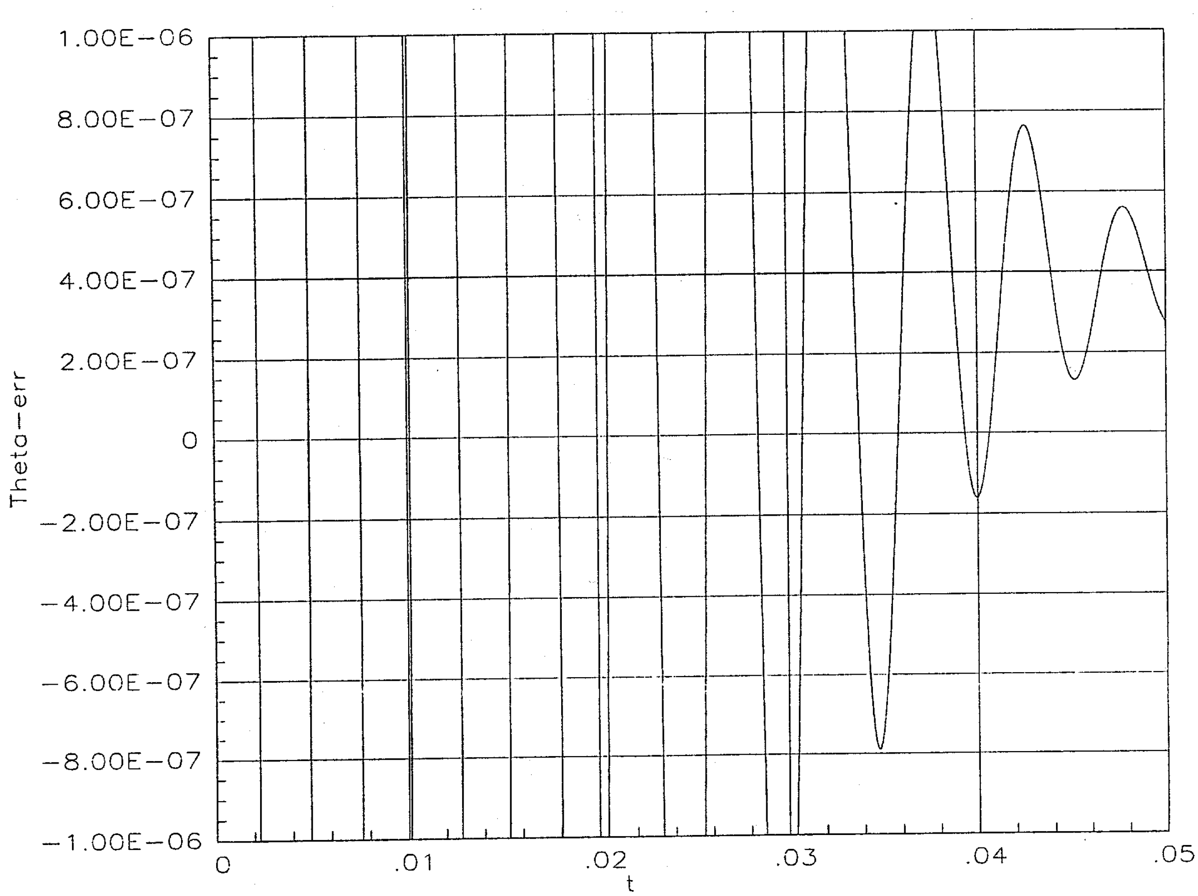


Figure 19. P-Gain Root Locus

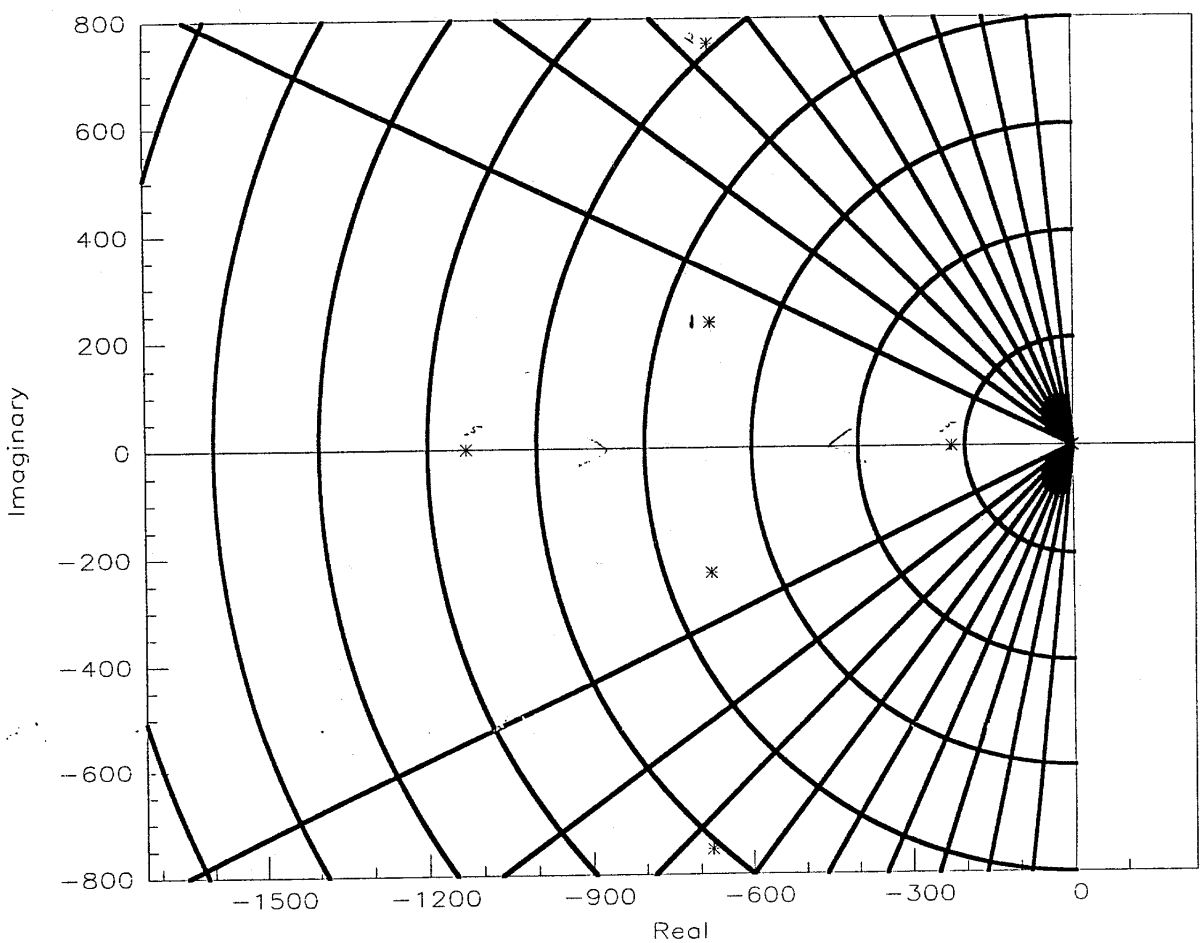


Figure 20. I-Grain Root Locus

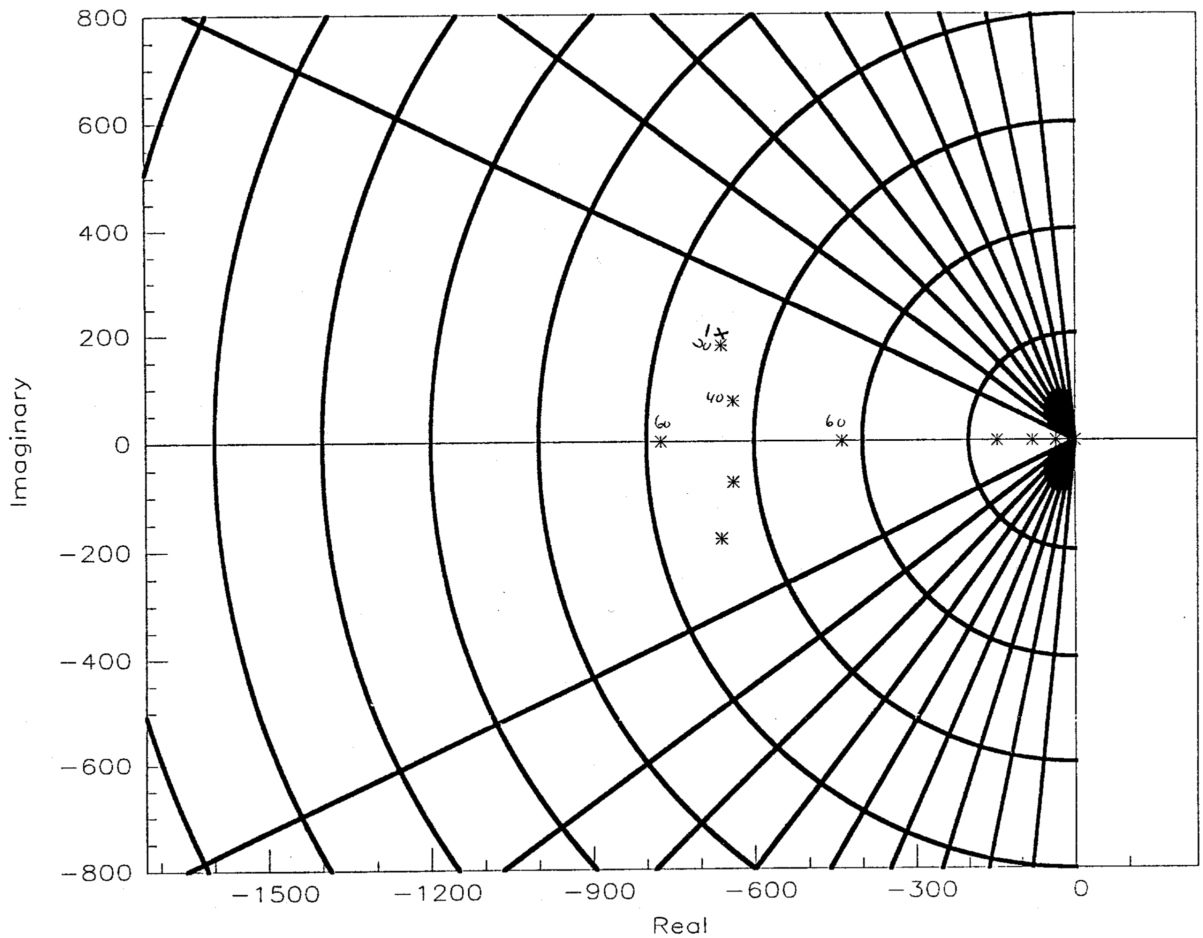


Figure 21. D-Grain Root Locus

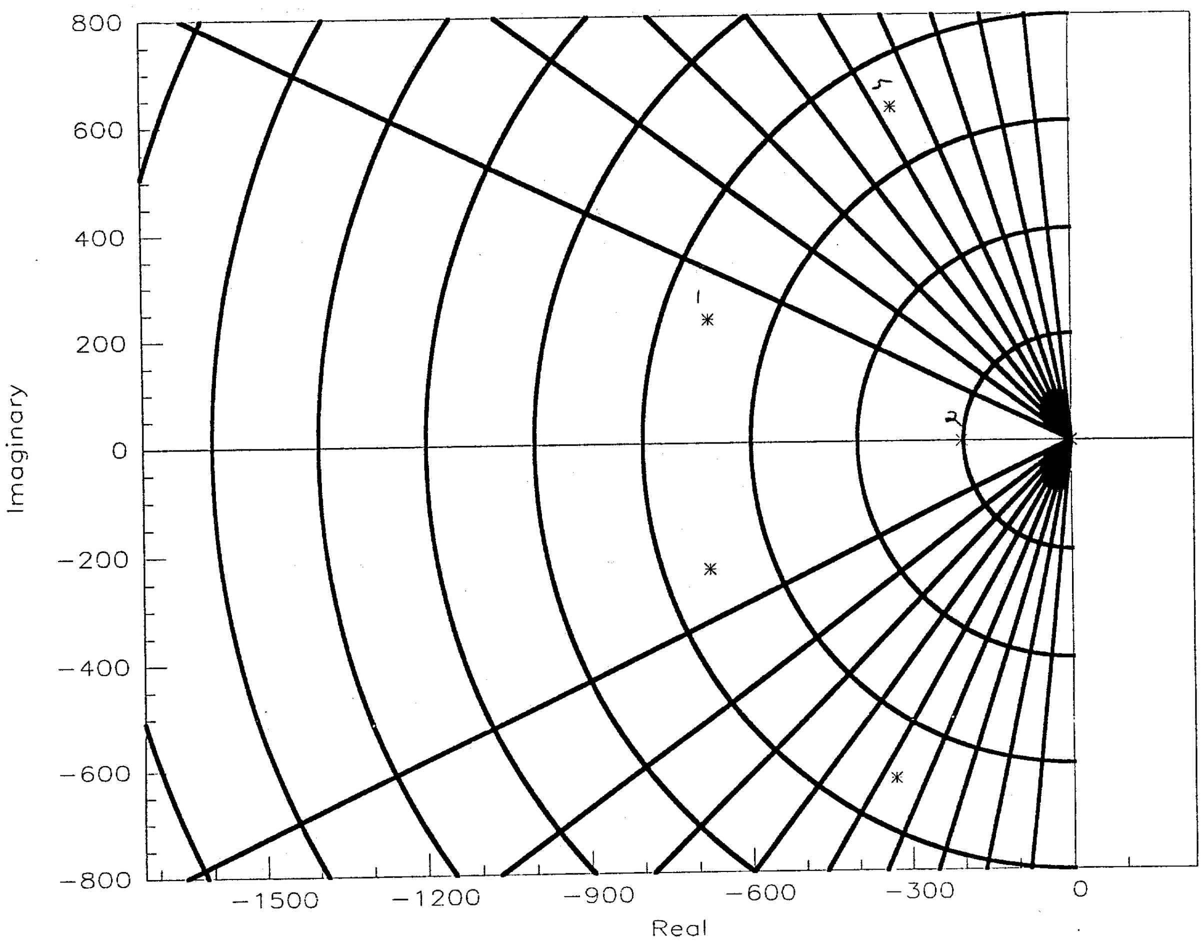


Figure 22. TD-Root Locus

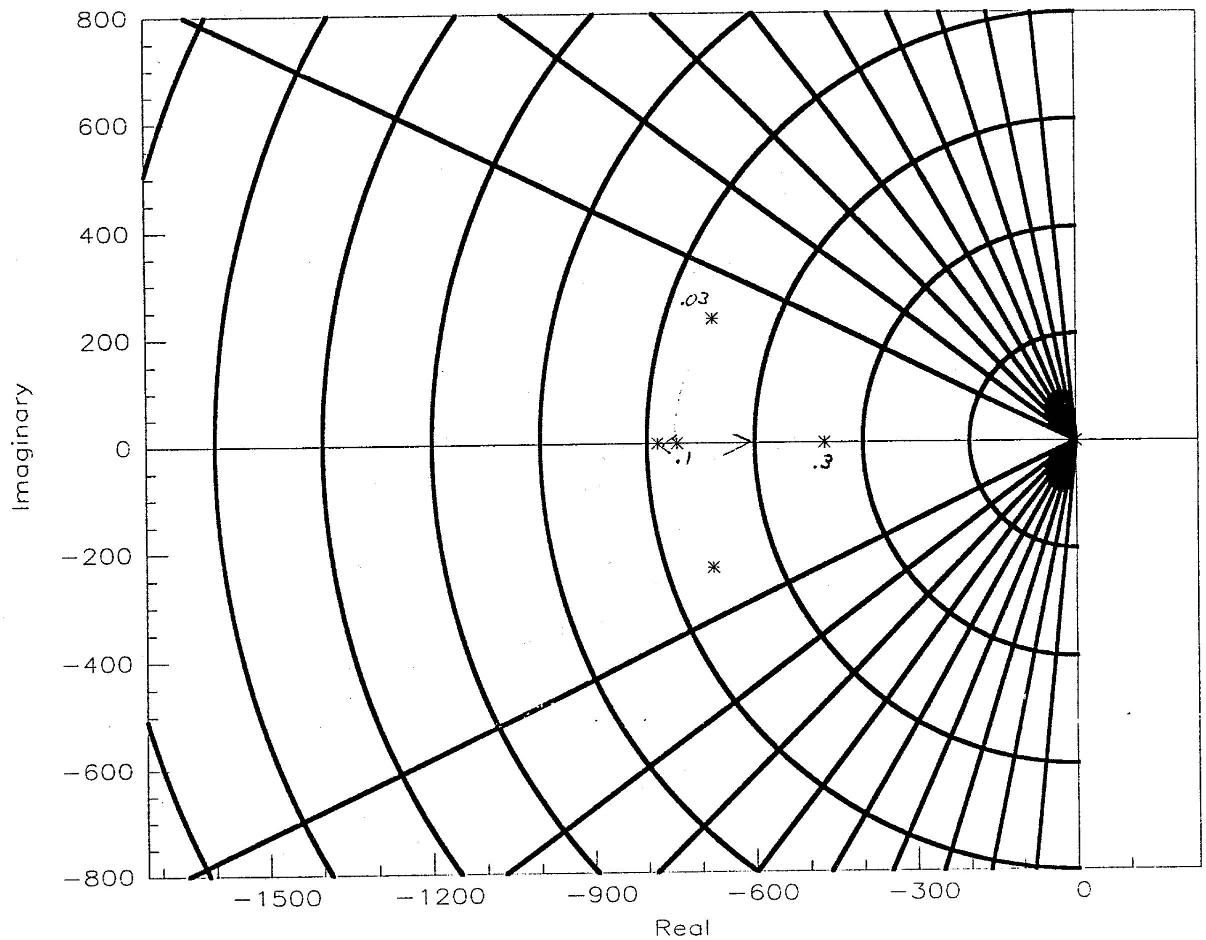


Figure 23. P-Grain Root Locus for TD=0.3ms

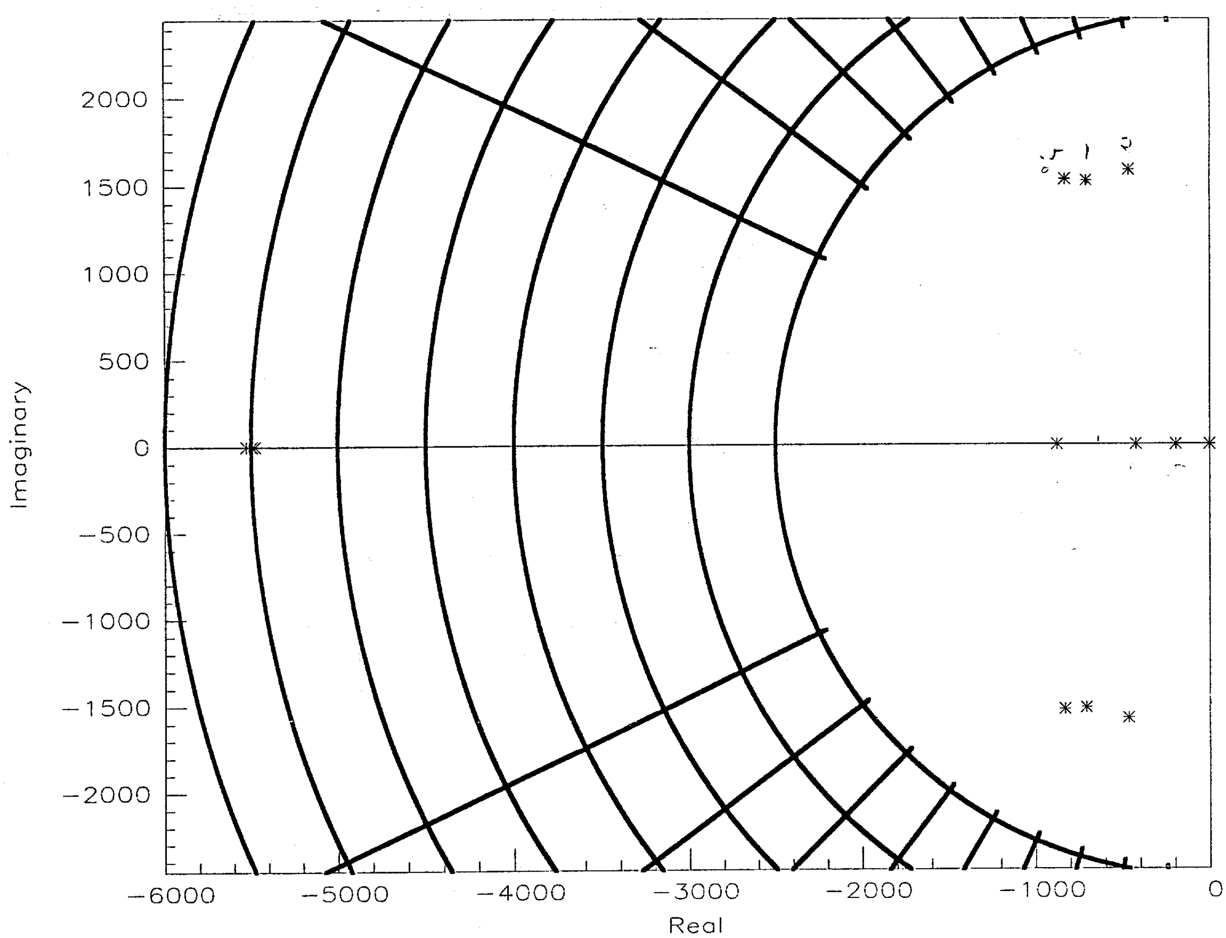


Figure 24. D-Grain Root Locus for TD=0.3ms

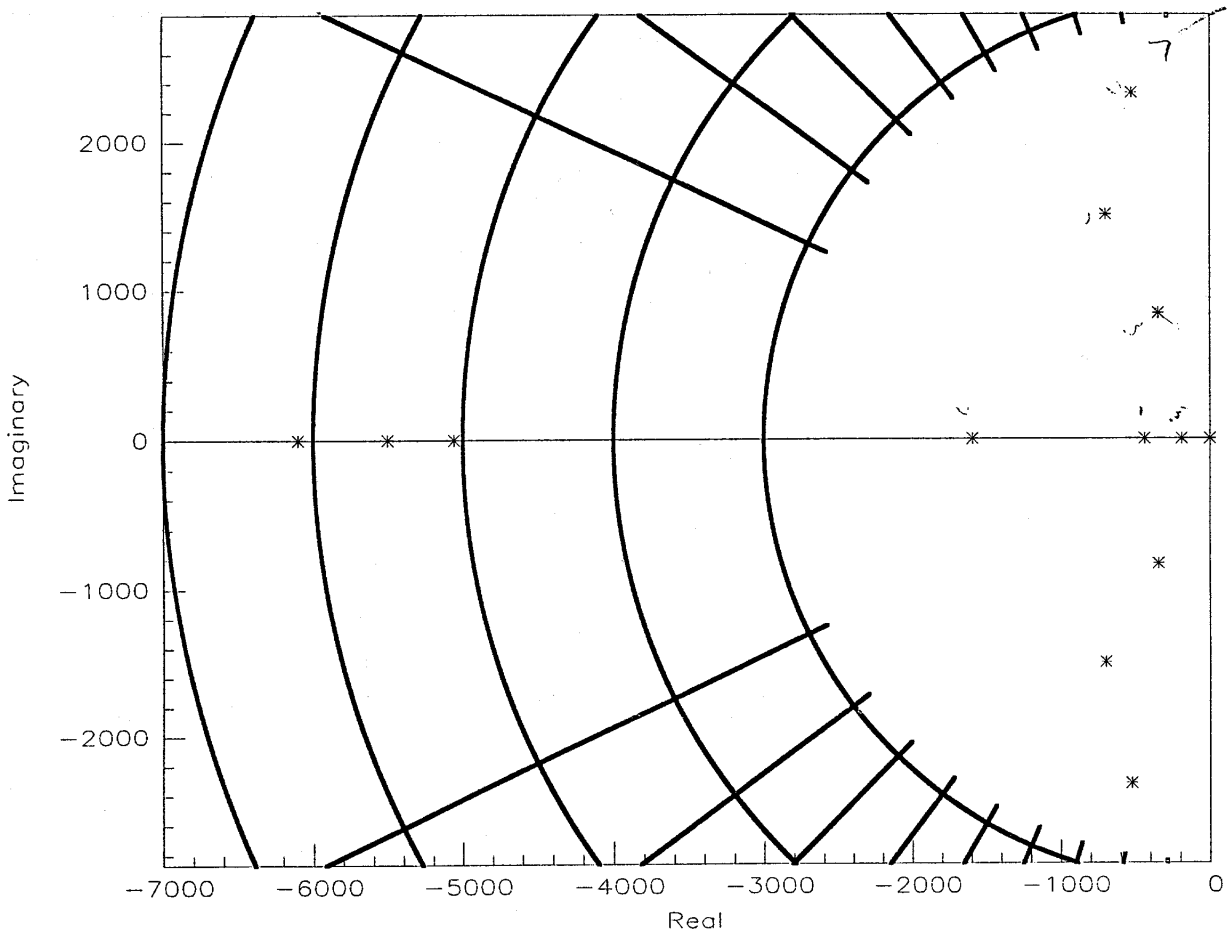


Figure 25. Simulation Results for TD=0.3, P-Grain=1.5P_o

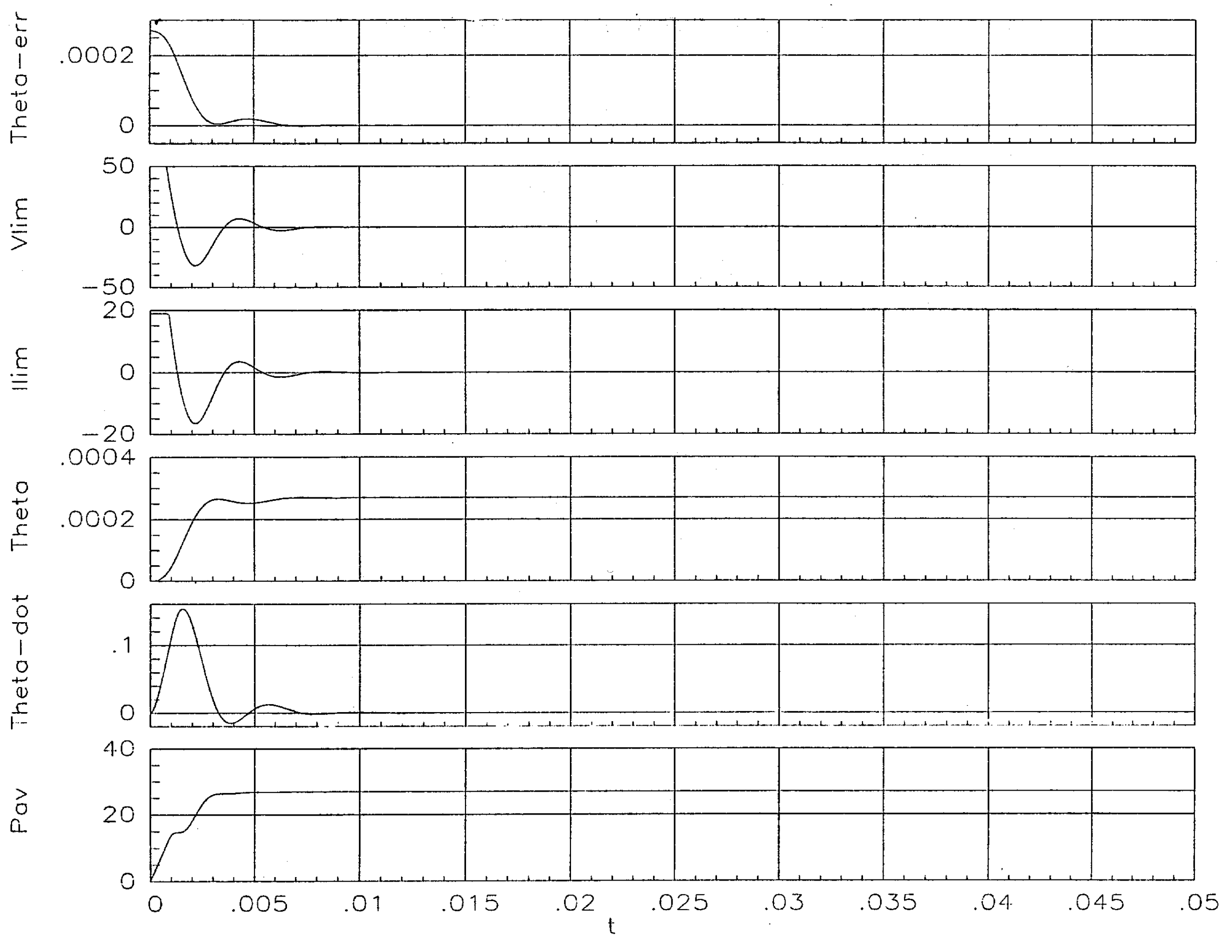


Figure 26. Error vs. Time for TD=0.3, P-Grain=1.5P_o

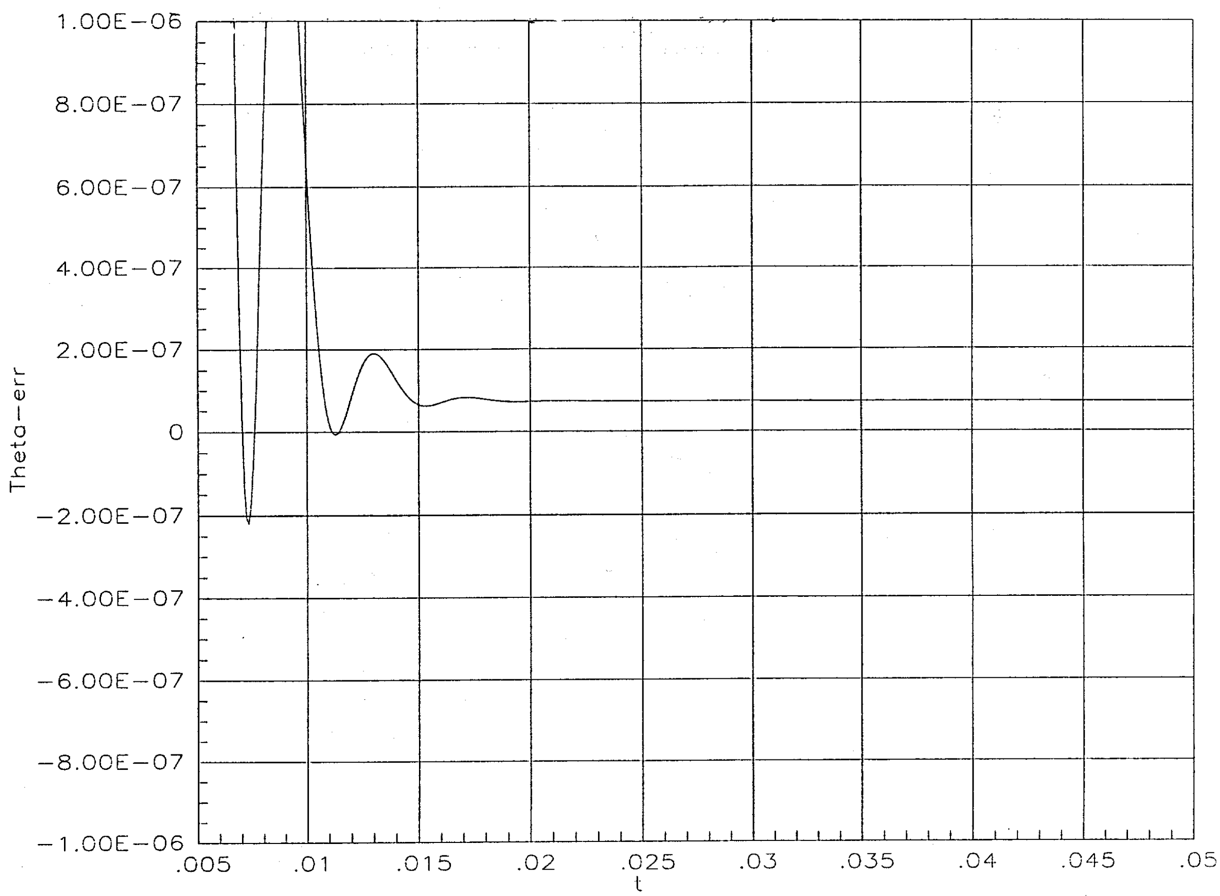


Figure 27. Root Locus for Discretized System

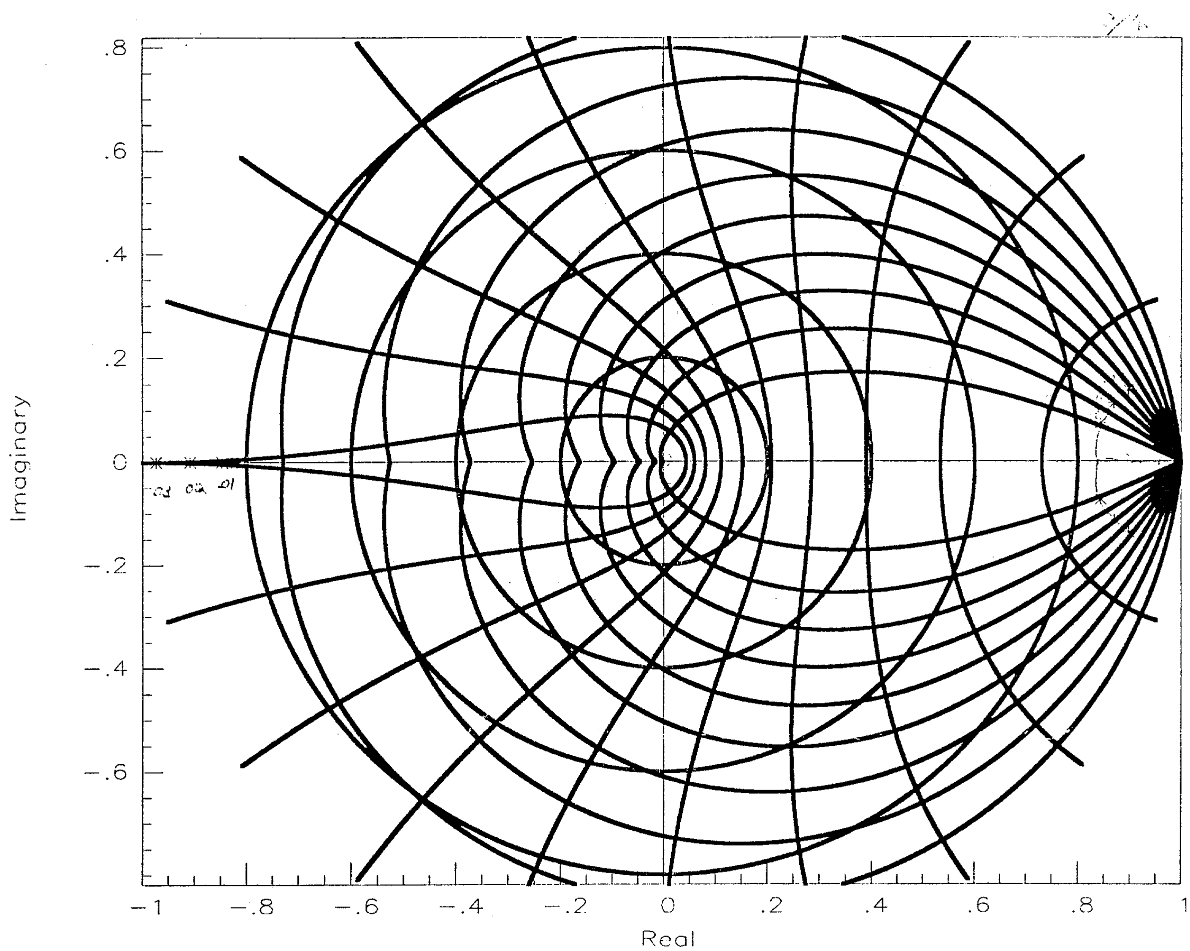


Figure 28. Root Locus for Baseline + Compensator 1

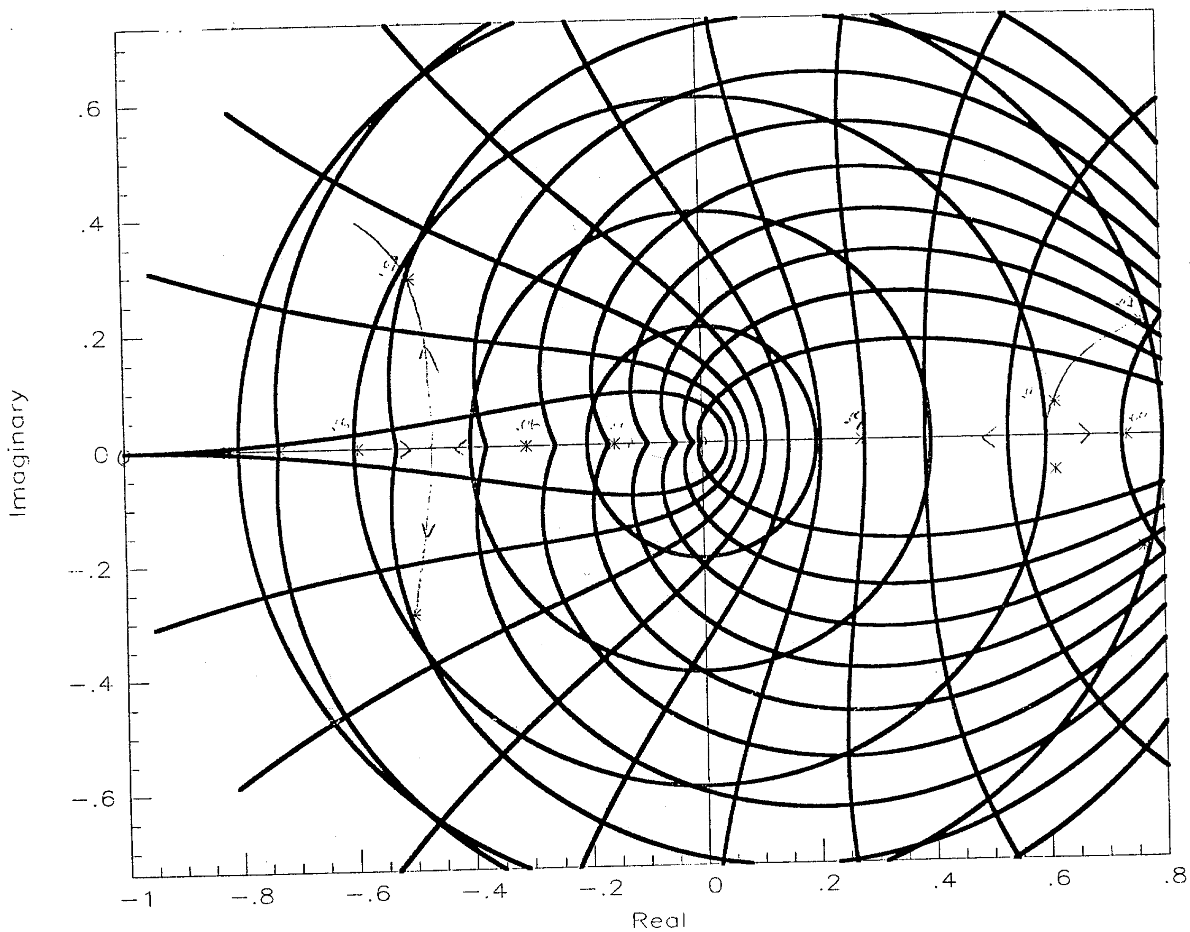


Figure 29. Simulation Results for Baseline + Compensator 1

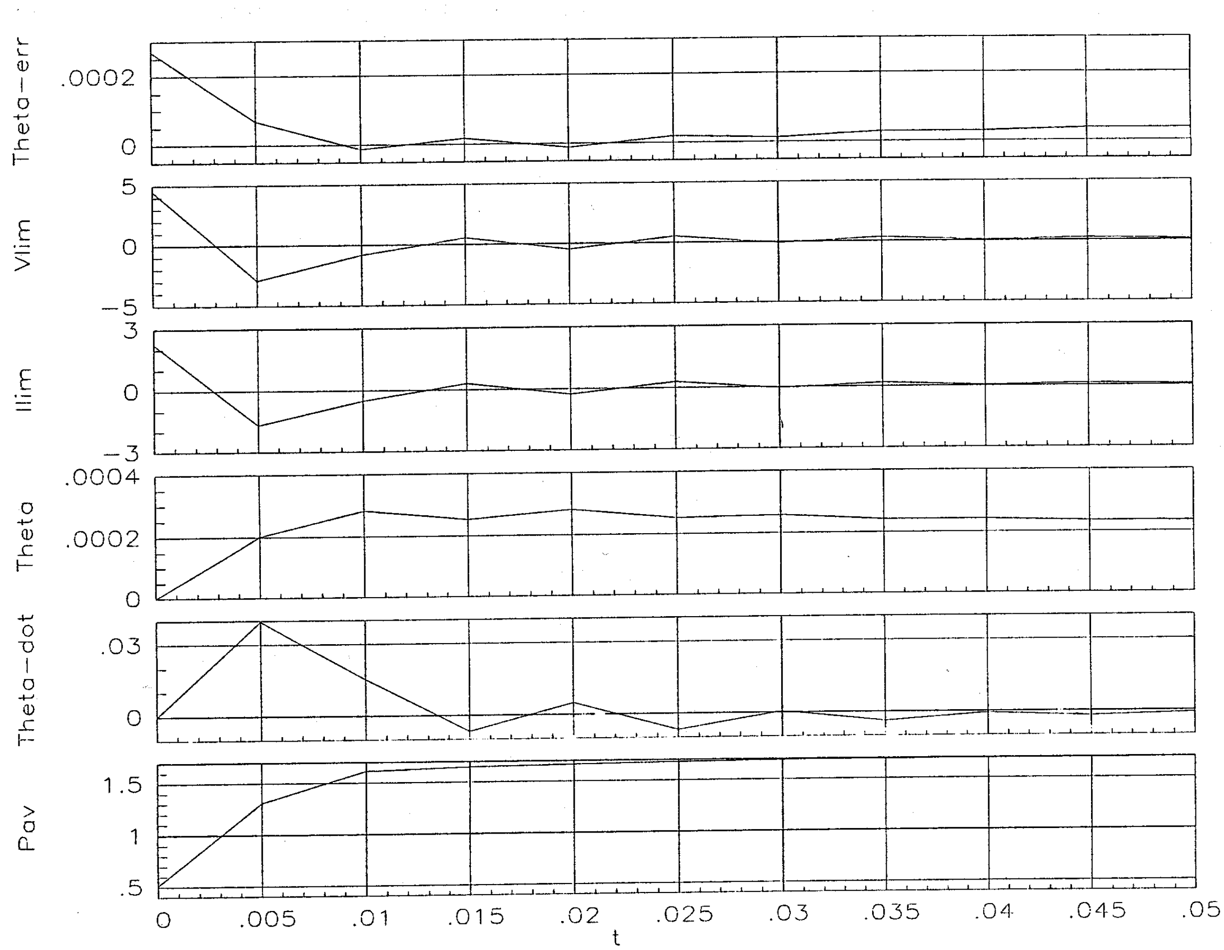


Figure 30. Block Diagram of a Bang-Bang + Linear Controller

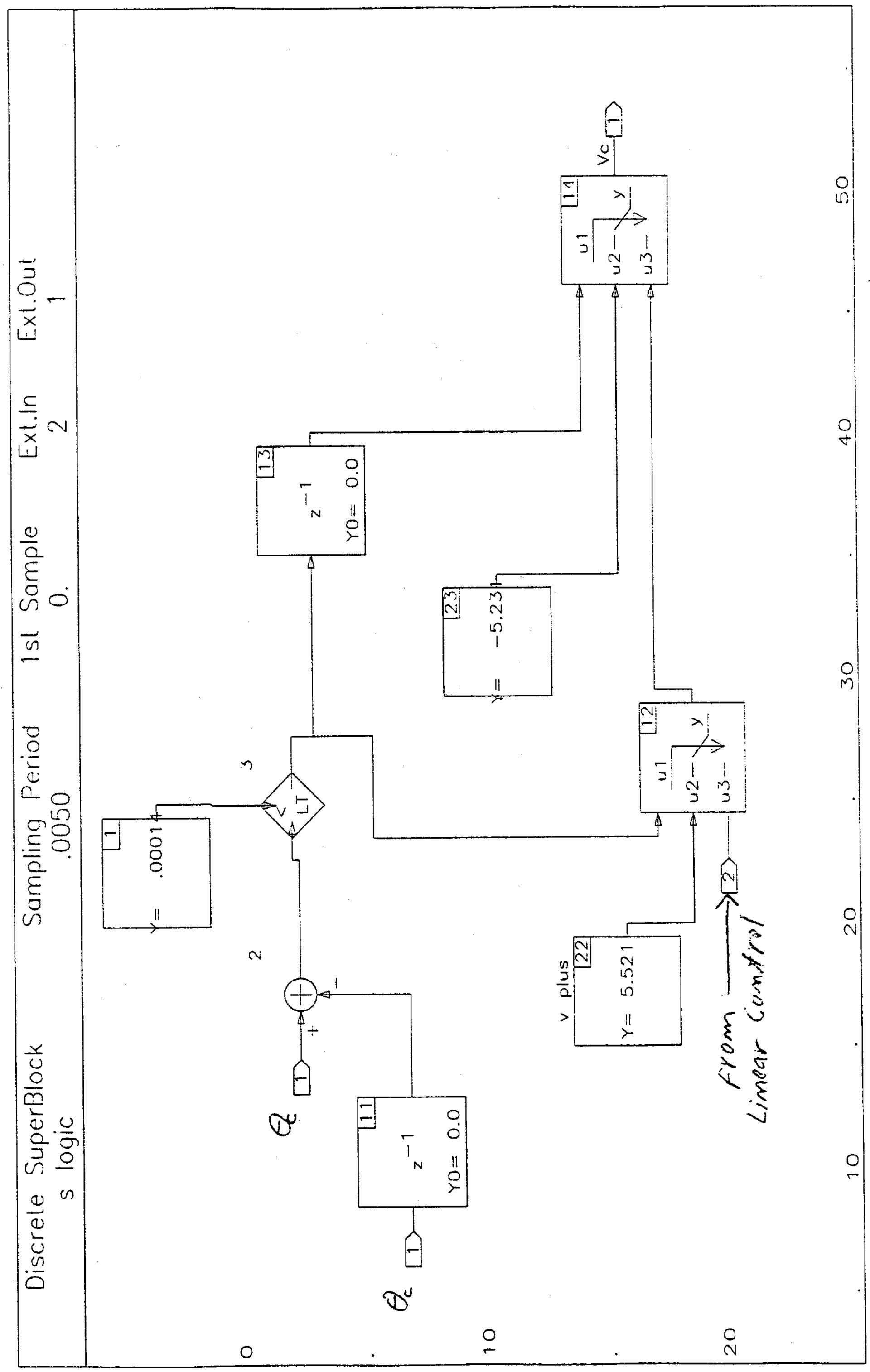


Figure 31. Simulation Results for Bang-Bang + Linear Controller

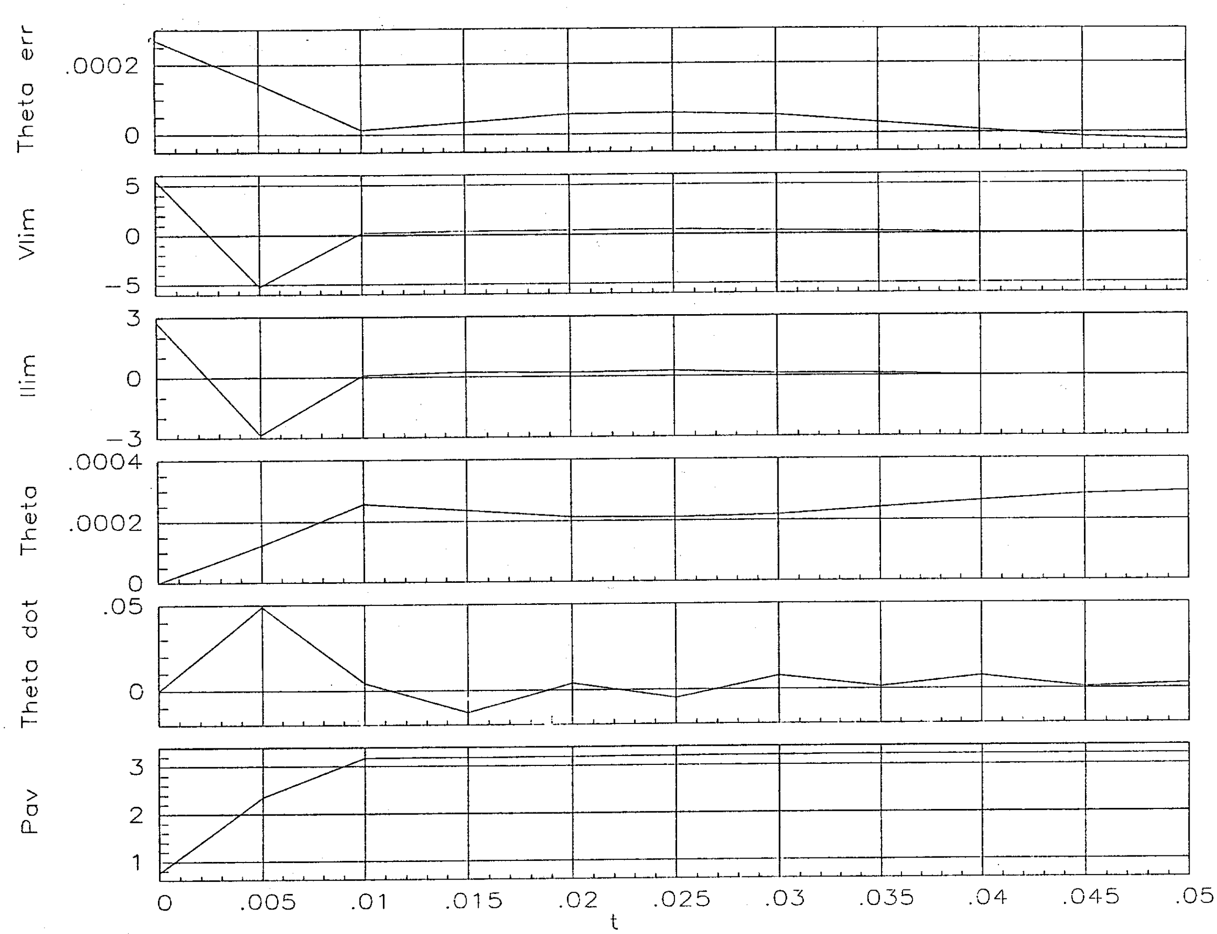


Figure 32. Simulation Results for Bang-Bang + Linear without VB or TE

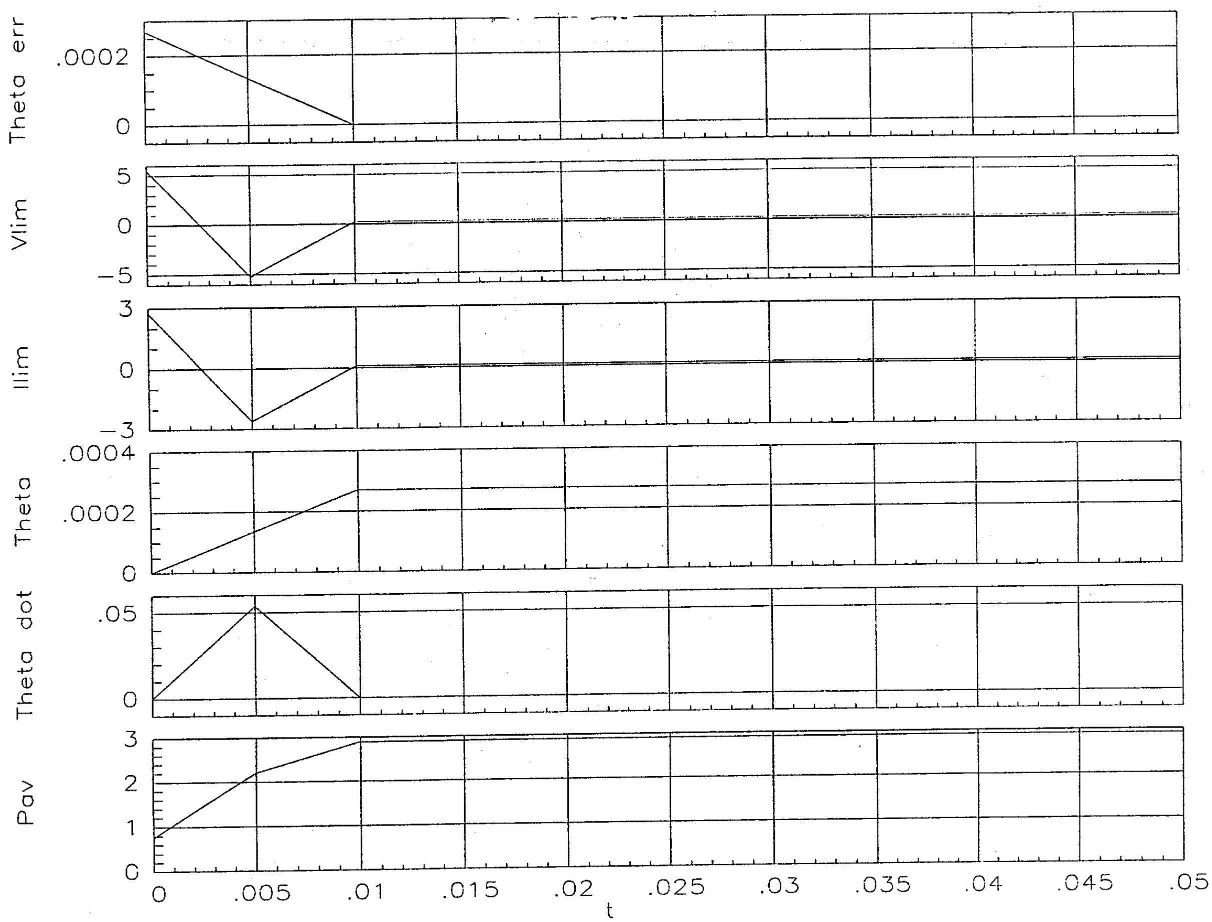


Figure 33. Kalman-Type Filter 1 Block Diagram

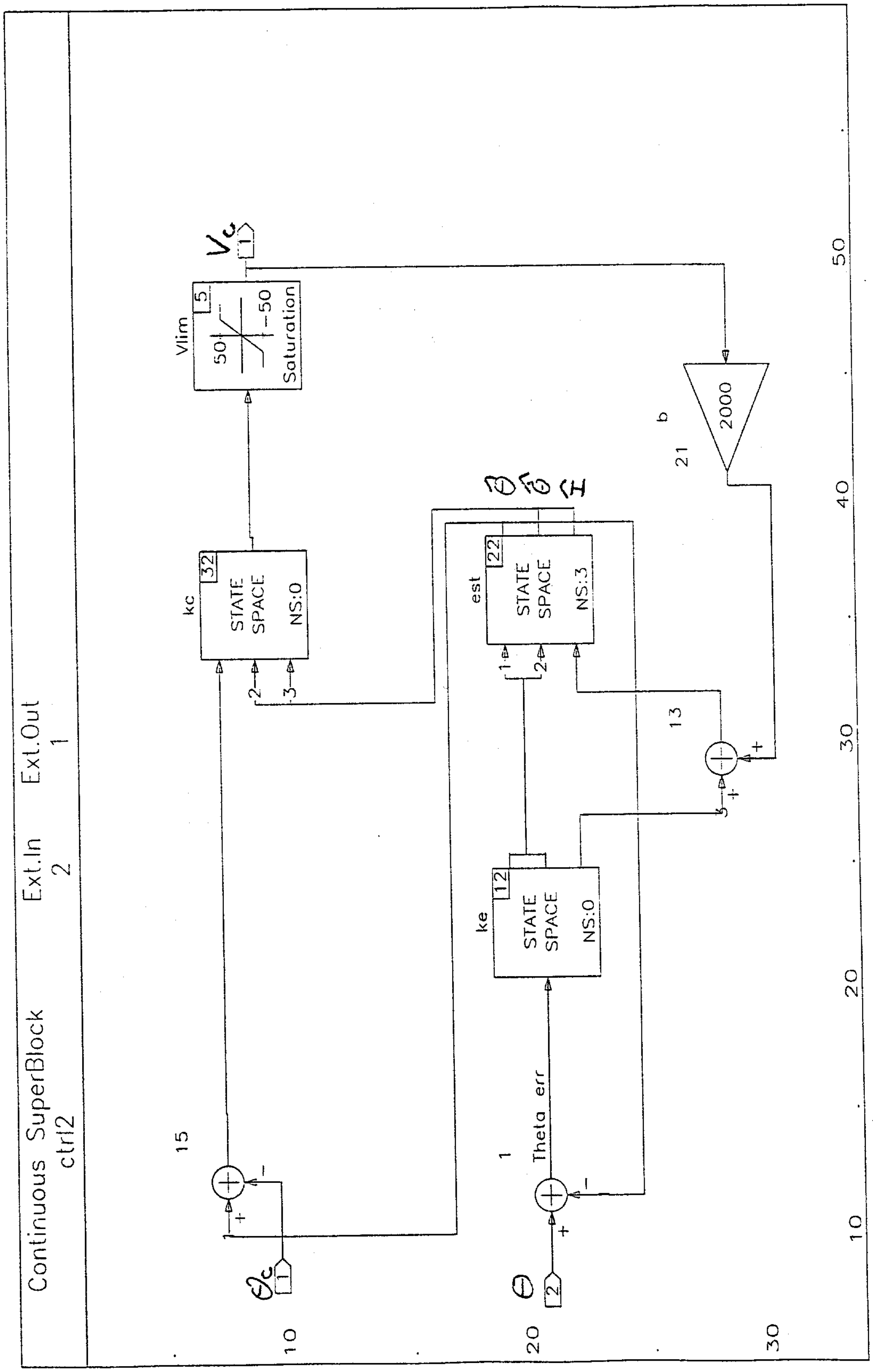


Figure 34. Kalman-Type Filter 1 Simulation Results

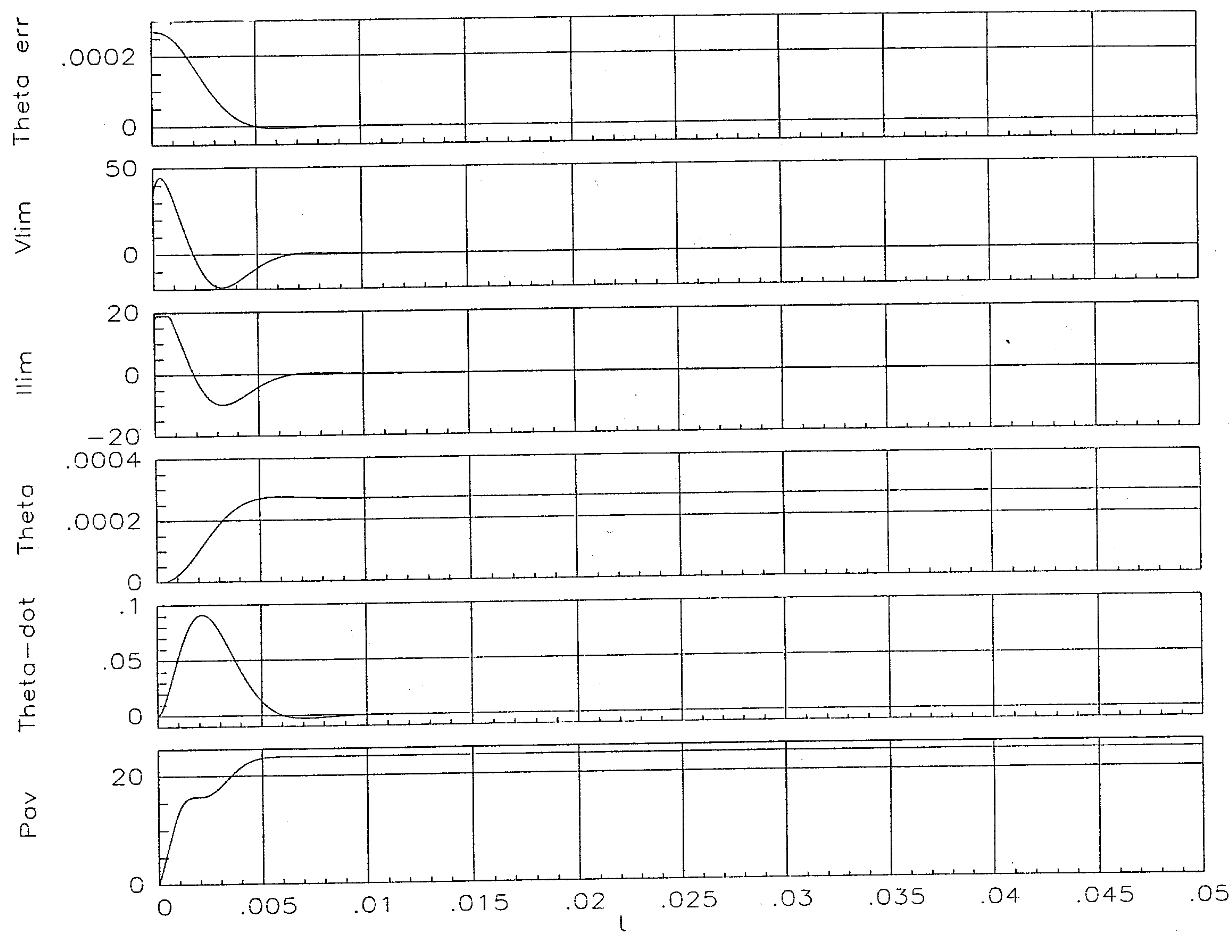


Figure 35. Kalman-Type Filter 1 Error vs. Time

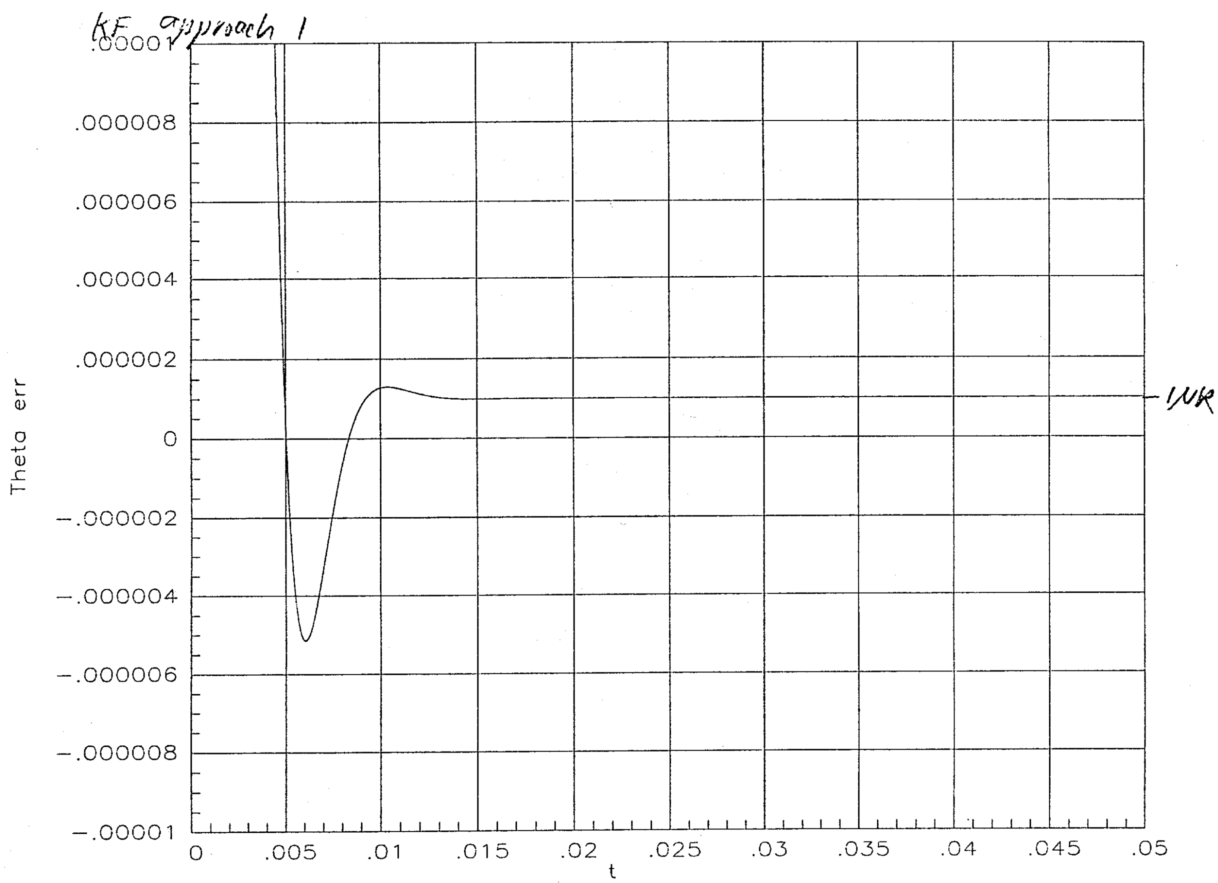


Figure 36. Kalman-Type Filter 2 Simulation Results

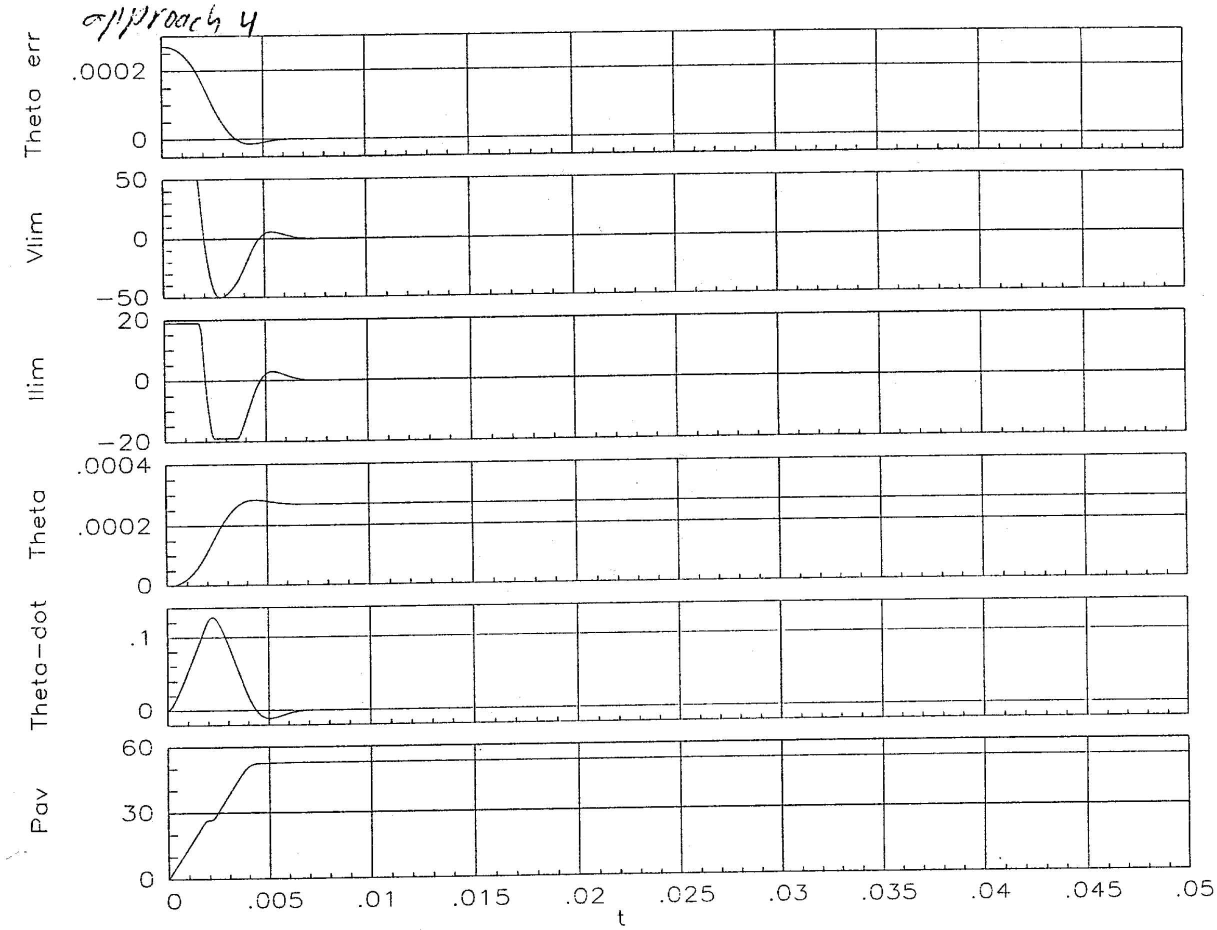


Figure 37. Kalman-Type Filter 2 Error vs. Time

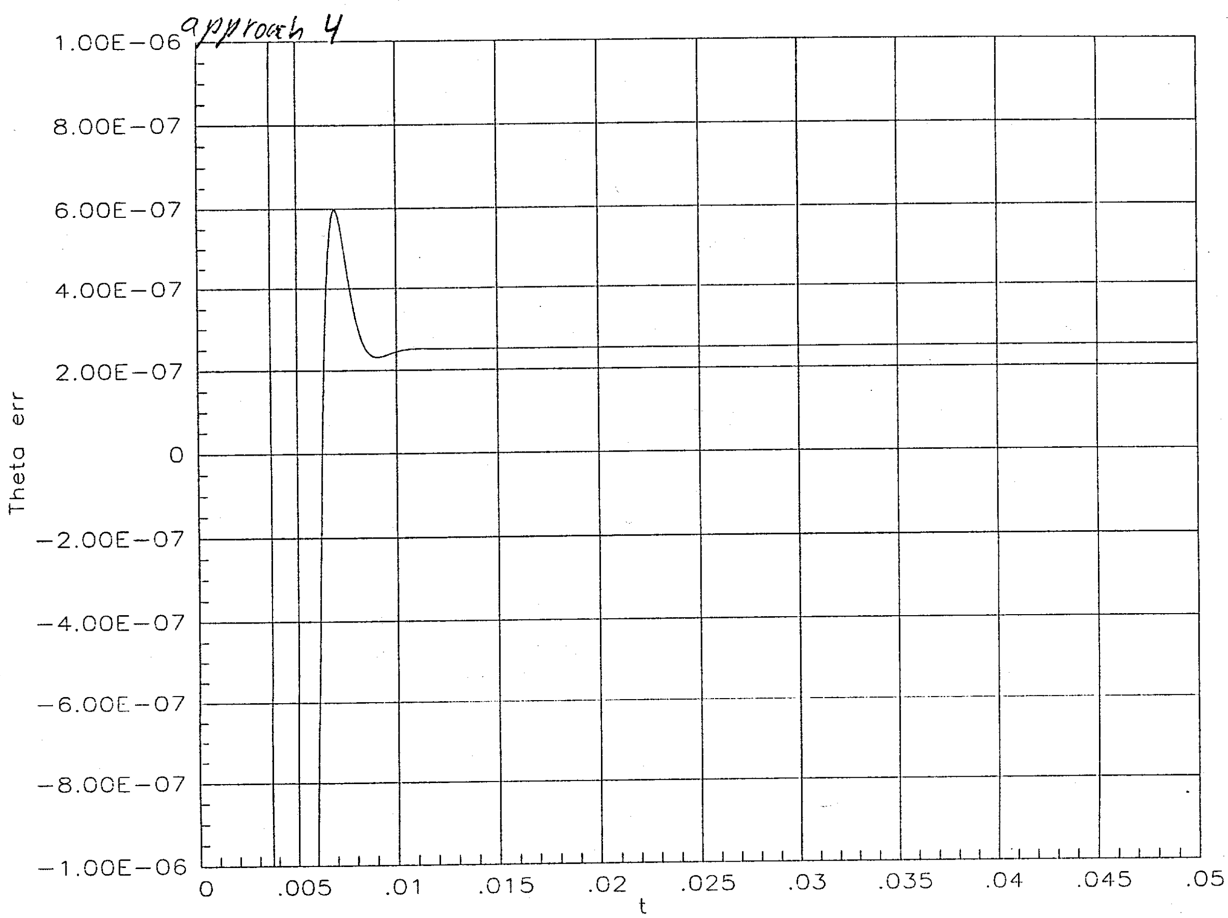


Figure 38. Integration Corrector Block Diagram

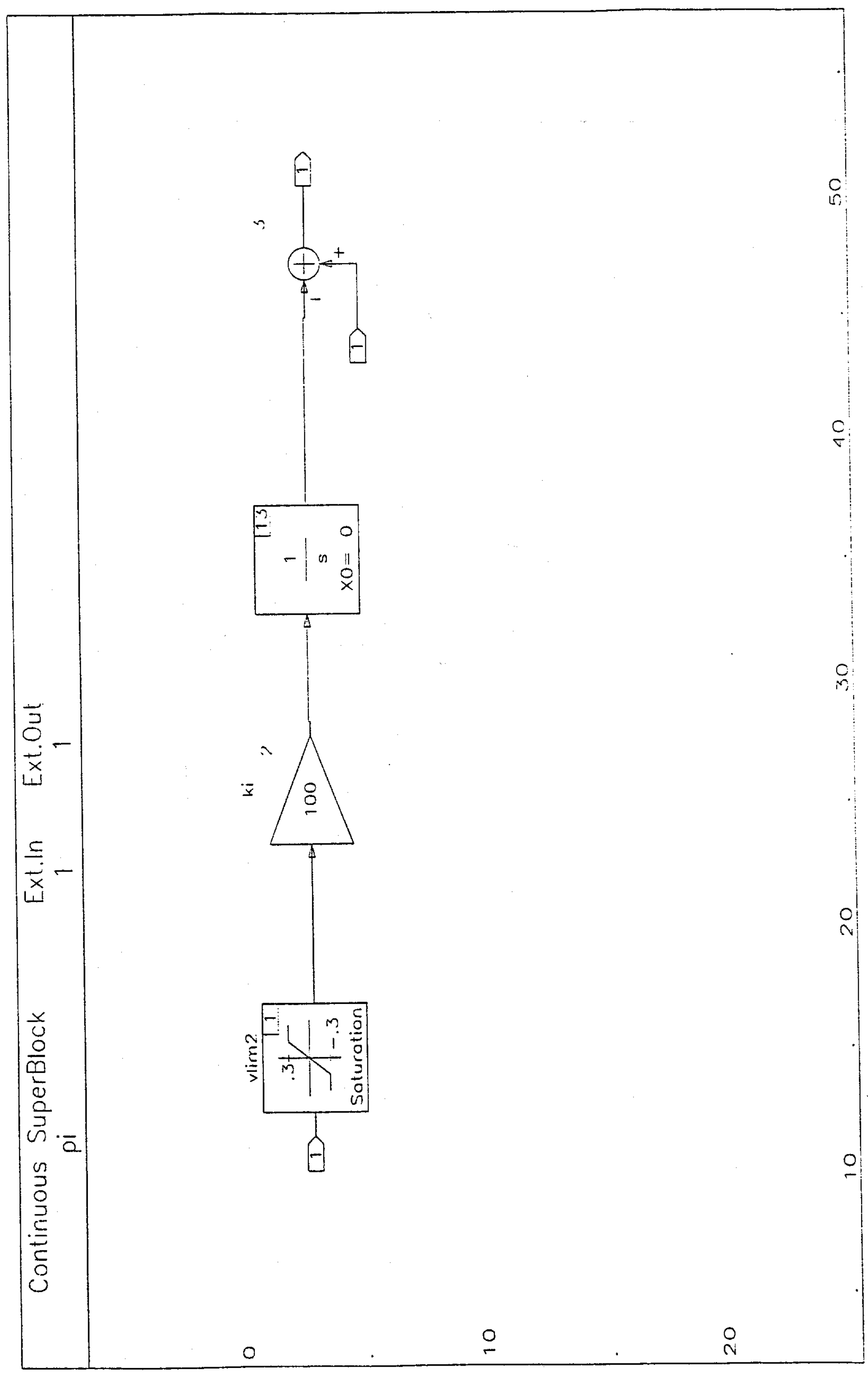


Figure 39. Kalman-Type Filter 2 _ Integration Simulation Results

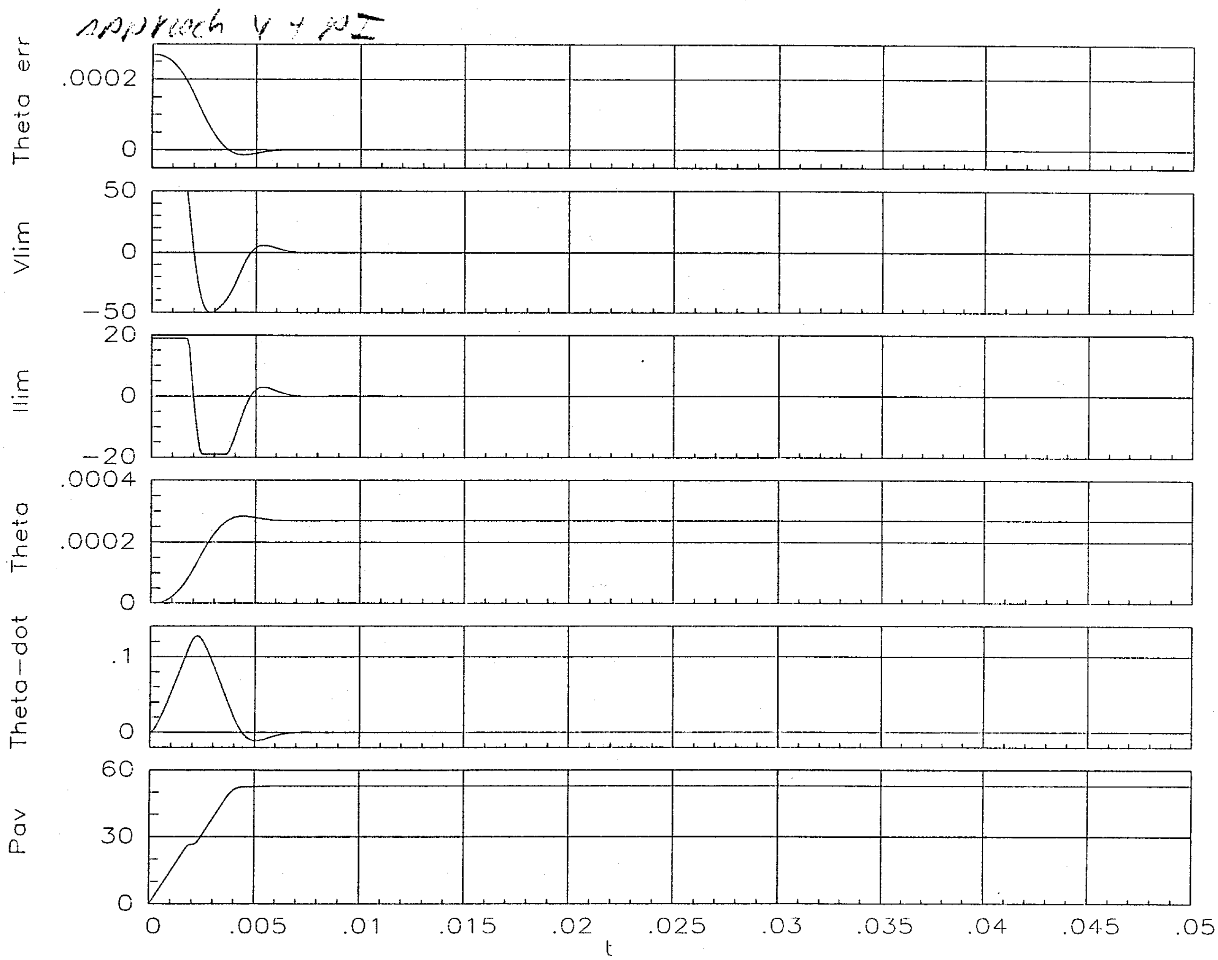
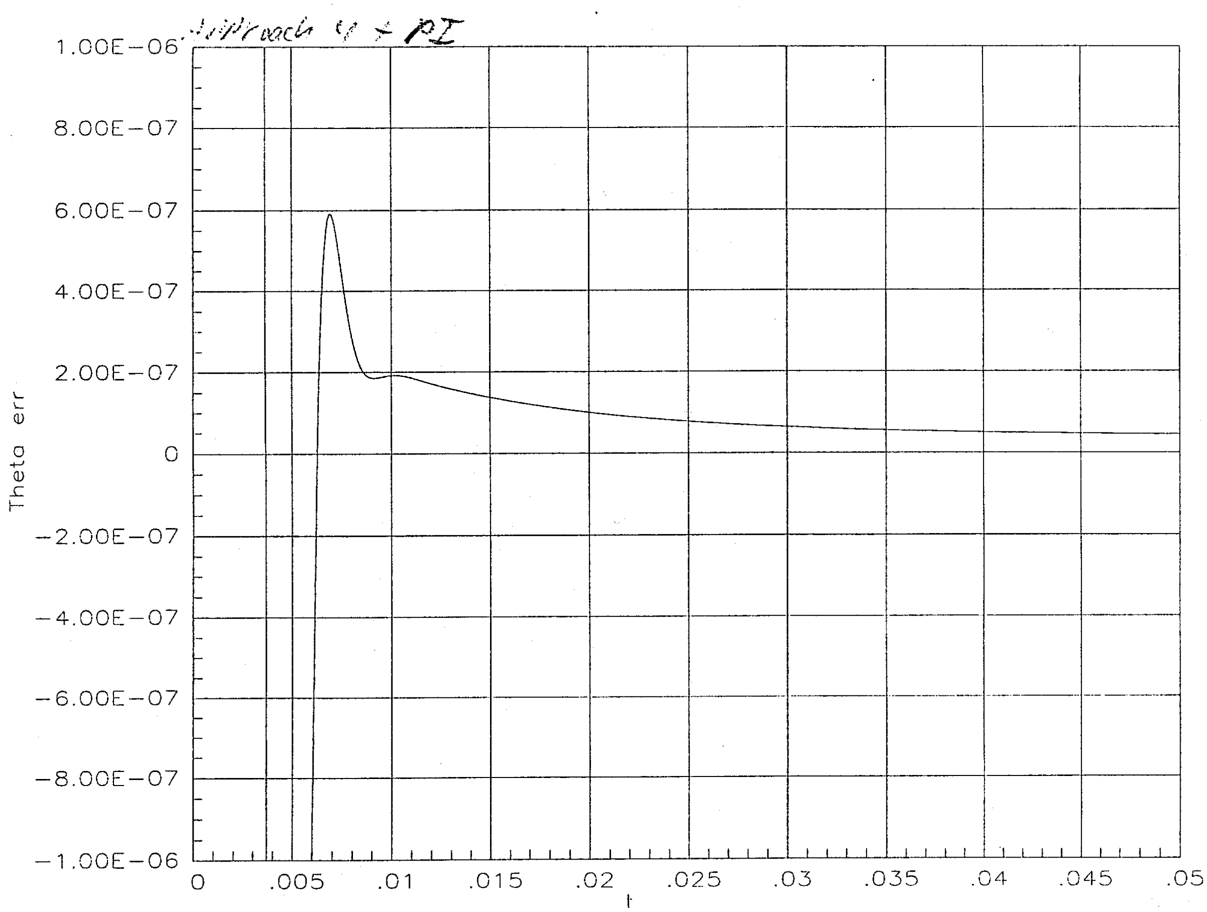


Figure 40. Kalman-Type Filter 2 + Integration Error vs. Time



APPENDIX I

GEMINI 8-METRE TELESCOPES PROJECT CONTROLS GROUP

To: **Distribution**
From: **Rick McGonegal**
Date: **January 17, 1993**
Subject: **Power Requirements for Chopper**

Summary
The power requirements for chopping have been estimated by servo simulation. The power required to meet specification during 10 Hz chopping with 15 arcsecond throw in the focal plane is:

Source	Power (watts)
chopper mechanism	2
momentum compensator	2
chopper sensors	1.5
momentum sensors	1.5
Total	7

Scientific Requirements
- 15 / 30 areseconds throw in focal plane at 10 / 5 Herz
- square wave chopping around optical axis
- 80% duty cycle

Engineering Requirements
- ± 28.2 / ± 56.4 arcsec motion of secondary mirror about CG at 10 5 Hz
- settle to ±0.10 arcsec about mirror CG within 10/20msec

Servo Requirements
- settling band of ±485 nanoradians within 10 / 20 msec for step size of 273/547 microradians

Chopper Parameters
- inertia about mirror CG is 4 Kg-M2
- 3 actuators placed 120 degrees apart at 300 mm from optical axis

Candidate Actuator Mechanism
The LA90-49 linear actuator from BEI is a voice coil actuator with a peak force of 667 N and a force constant of 35 N/ampere. The specification sheet is attached.

Assumptions
- mirror chopper mechanism mechanical damping is -3%

- ignore chopper mechanical resonances
- servo system is effectively critically damped

Power Estimate From Electromechanics
It is possible to estimate the power based on electromechanical arguments.

The optimum strategy to move an inertia from one angular position to another in minimum time demands maximum acceleration for half the way, followed by maximum deceleration to a full stop. It can be shown that the total travel time is:

$$T = 2*\mathrm{sqrt}(\theta*I/Tmax)$$

where T is the time to travel from peak to peak
θ is the peak to peak angle the secondary moves
I is the inertia of the secondary
Tmax is the maximum torque

For a chopping secondary with T and θ as specifications and I as a constraint then one uses the above relation to calculate the torque required. The electromechanical driver must supply this torque and in turn, the power amplifier must supply the current to the driver.

In our case we have 10 / 20 msec and 273 / 547 radians for T and 1=4 Kg-m^2. This leads to values of Tmax of 43.68 / 21.88 Nm.

This torque will be supplied by N electromechanical drivers located a distance I from the axis of rotation, each providing a force Fmax.

$$Tmax = N*1*Fmax$$

In the Gemini design there are 3 drivers located 120 apart and 0.3 meters from the axis of rotation. This leads to each electromechanical driver being capable of delivering 48.53 / 24.31 N.

For the candidate actuator this translates to a current of 1.38 / 0.69 amps (U = force constant 35 N/amp).

The average power can be calculated as

$$Pavg = 2*f*t*Pmax + (1-2*f*t)*Pss$$

where f is chop frequency in herz
t is 10 / 20 msecs
Pmax is power during transition
Pss is steady state power from holding the mirror in place against spring tension

Pmax can be calculated as

$$Pmax = N*Imax^2*R$$

which in the Gemini case is 17.1 / 4.28 watts.

The torque required to hold the mirror in position at either extreme must be supplied by the N actuators.

$$K*\theta/2 = N*1*U*Iss$$

which in the Gemini case leads to Iss = 0.028 / 0.056 amps. Pss can be calculated from

$$Pss = N*Iss^2*R$$

which yields 7 / 28 milliwatts.

From this the average power **is** 3.43 / 0.878 watts for the 10 / 5 Hz cases. If we assume that the devices are 50% efficient, that the momentum compensation system is identical, and that a total of 6 position sensors at 500 milliwatts each are required then the total power consumption is 16.72 watts.

Power Estimate From Servo Simulation
The servo simulation was designed to meet the settling requirements and then, using the candidate actuator, the power requirements were calculated from the simulation output.

The transfer function for the secondary mechanism was assumed to be a second order system represented by

$$\theta/Torque = (Js^2+Fs+K)^{-1}$$

From the chopper parameters and assumptions we have

J = 4 Nm
F = 10 Nm/rad/sec
K = 6500 Nm/rad

The actuator transfer function was assumed to be

$$Torque/current = U*N*l$$

where U = force constant = 35 N/amp
N = number of actuators = 3
1 = lever arm of actuator = 0.3 m

The peak current capability of an individual actuator, 19 amps, was enforced by including a current saturation block at the input to the actuator transfer function.
The simulation setup is illustrated in the attached figure. A simple position feedback with PID controller was used for this initial estimate. The PID gains were set by increasing the proportional gain until the rise time requirement was met, increasing the derivative gain until the settling requirement was met, and then increasing the integral gain until the steady state error was tolerable. This yields values of

P = 6.25e4
I = l.0e5
D = 165

The verification that this design meets specification is shown in the attached figure - the mirror position is within the 485 nanoradian settling band after 9.3 milliseconds.

The simulation was run for 0.05 seconds, equivalent to one half chop cycle at 10 Herz, and the instantaneous power measurements were collected, along with the time, in a file. This file was then integrated to calculate the average power over a chopper cycle. The instantaneous torque, current, and power curves are shown in the attached graphs. The cumulative integral is also shown over a servo cycle.

The simulation gives a result of 0.98 watts average. If one assumes that the momentum compensation system requires the same power and that the systems are 50% efficient then the power per system is 2 watts for a total of 4 watts. The system will require a maximum of one position sensor for each actuator for a total of 6 - we have allocated 500 milliwatts per sensor.

Caveats
The power calculation above does not take account of the tip/tilt requirement or the effects of wind during chopping. The settling band specification must be maintained in the presence of these external disturbances and the current plan is to use the tip/tilt capability of the mirror to do so. Given that the peak to peak amplitudes of these effects is an order of magnitude smaller than the ebopping amplitude there should be an insignificant increase in the power required.

The Next Step
In order to improve on both the power estimate and on the servo design required to meet specification more detailed models should be explored. This next step should be taken once a detailed FEA model of the chopper mechanism is available. The next step requires:

- FEA model of chopper
- more detailed model of actuator and sensor
- include wind and atmospheric tip/tilt as disturbance inputs

include maximum torque limits imposed by secondary support structure and upper end