Primary Mirror Forces from a Distributed Hydraulic "Axial" Support System

Introduction

One can imagine that a thin meniscus mirror 8.2m in diameter will deform easily when subjected to forces. These forces are due to such factors as changing gravity orientation, wind loads, active and support forces, thermal expansion, etc. It is the role of analysis (analytical, finite-difference, finite-element, etc.) to predict the mirror surface distortions for a given set of boundary conditions; imposed displacements along with internal and external forces. This report deals only with predicting forces on the mirror, not with the resulting distortion due to these forces.

A number of simplifying assumptions will be used for this report. This should make the material easier to follow while not sacrificing the basic ideas necessary for an understanding of such a system. The actual analysis for the telescope will not always use the same assumptions and will be somewhat more complicated.

First we will simplify the geometry for menisus mirrors by neglecting the curvature and the central hole. **Figure 1** shows the simplified mirror geometry. The mirror surface is indicated and we assume that no support will be allowed on this surface. The telescope is Alt-Az, and the mirror rotates relative to the gravity vector about the x axis. The x-y-z coordinates are fixed to the mirror.

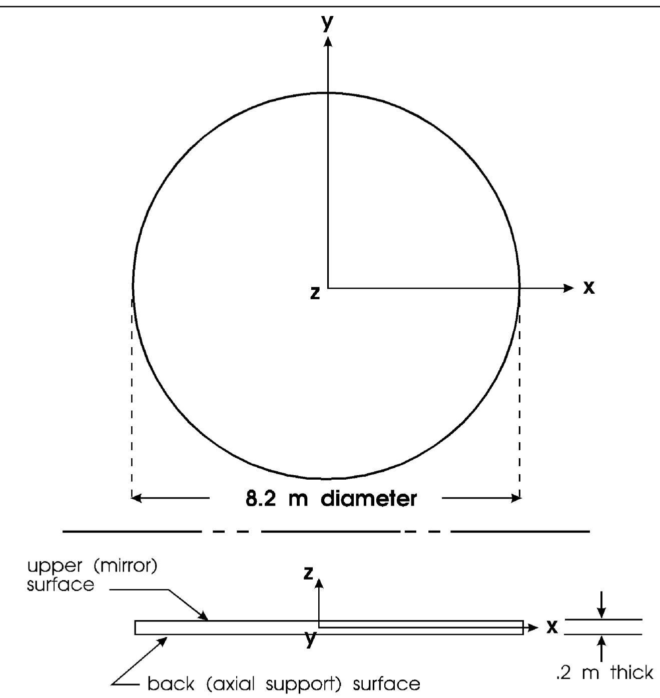

Figure 1. Simplified geometry for the meniscus mirror.

The mirror is shown in **Figure 2** at some zenith angle, θ. The range of θ for the telescope is 0 ≤ θ ≤ 75°. The weight of the mirror is called W, and W ≈ 225,000 N or 50,000 lbs. The weight-gravity vector, $\vec{W}$, is shown and has a resultant at the geometric center of the mirror ("c.g"). For convenience, we can resolve into its "axial" component, W cosθ, and its "lateral" component, W sinθ. Some system of forces must support these weight-gravity forces along with any other forces applied to the mirror. We will assume that forces on the back surface (Figure 1) will take care of W cosθ as well as any other forces in the axial direction. These are the "axial supports" discussed here. The lateral forces, W sinθ, along with any other force perpendicular to the axis will be taken care of by a "lateral support" system. These lateral supports will be assumed but not discussed here.

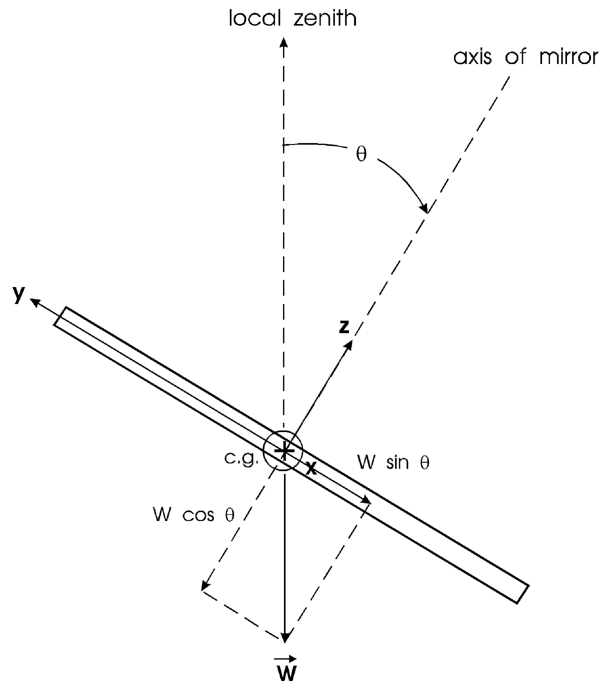

Figure 2. The mirror in relation to the gravity-weight vector, W.

At zenith about 80% of the axial weight component, W cosθ, will be supported by a uniform air pressure[1] over the back surface. The other 20% of the weight plus any external loads will be resisted by a distributed hydraulic system. The forces applied to the mirror through this hydraulic system are described by this report. However, not only must the axial supports resist these loads, but they must do it in such a way that the mirror surface does not distort past its specification (poor "figure"), nor does it tilt or piston (rigid body motions) past some specified amount.

[1] Myung Cho and Ronald Price, "Optimization of Support Point Locations and Force Levels of the Primary Mirror Support System" (RPT-O-G0017), October 1993.

Some of the forces will change with time. Examples are: slewing the telescope, wind buffeting of the mirror, vibrations in the telescope structure, etc. These time-dependent values, both fast and slow variations, will not be addressed here. Only static, or "quasi-static", with a bandwidth of less than .02 Hz will be assumed.

A review of the major assumptions are:

(a) Boundary conditions (displacements and forces) are predicted, but no resulting mirror distortions are presented.
(b) The mirror geometry has been simplified to a flat meniscus with no central hole.
(c) Only "axial" loads are considered. The hydraulic units to be described carry only ≈ 20% of the axial weight and the external axial loads.
(d) Only static forces are analyzed.

A. Idealized static cases

Case 1. 3-point support at 120° intervals and radius r.

There is an axial support at each of the positions shown in **Figure 3**. The (x,y) coordinates are shown in terms of the radius r and the angles. At each of these "points" there is a hydraulic unit that might look like the unit sketched below (**Figure 4**).

The load cell measures the force applied to the mirror. The piston is on a roll diaphragm ("bellowfram") which is sealed with a fluid inside. The fluid has a weight density, g, and is "incompressible". Neglect the weight of the load cell (L.C.) and piston, and assume the diameter of the unit is very small compared to the mirror size. The "effective area" of each unit is A_i (i=1, 2, 3 the unit number). In practice, this area is intermediate in size between the piston area and the area defined by the wall constraining the fluid. Then the force output read by L.C. is related to the pressure and area by

$$F_i = A_i\, p_i$$

where p_i is the fluid pressure in the sealed unit.

Reading the output of the load cell, or measuring the pressure, p_i, would require the hardware involved. We can predict the forces from static principles once the axial weight component, W_a is known. For static equilibrium we know that

$$F_1 + F_2 + F_3 - W_a = 0$$

The moments about the c.g. are also zero: therefore,

$$F_1\, x_1 + F_2\, x_2 + F_3\, x_3 = 0$$
$$F_1\, y_1 + F_2\, y_2 + F_3\, y_3 = 0$$

Using the x,y components given; then

$$F_1 + F_2 + F_3 = + W_a$$

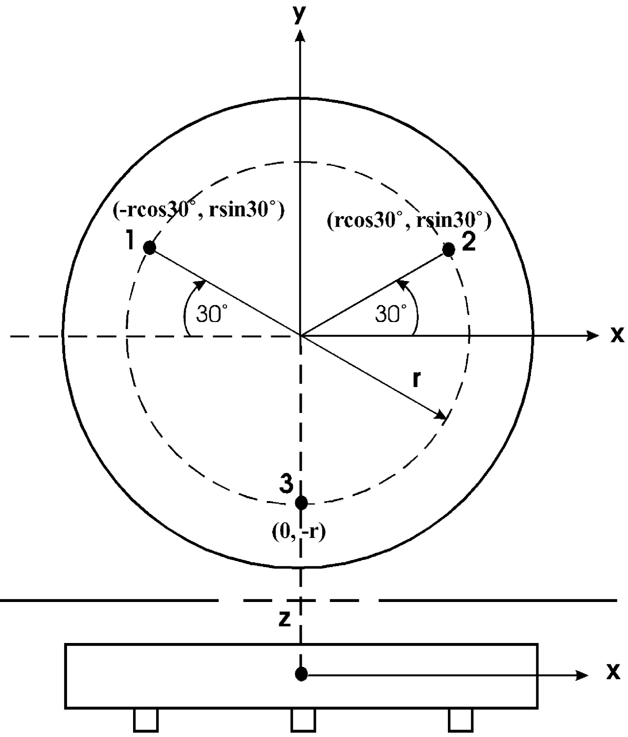

Figure 3.

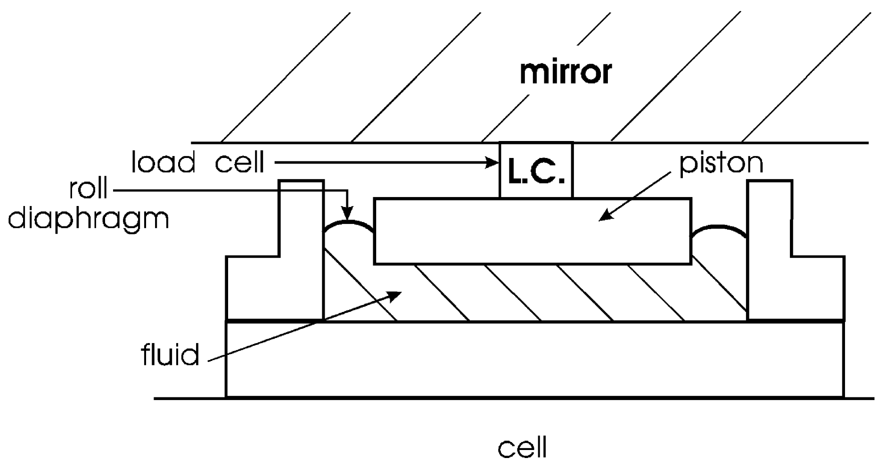

Figure 4.

$$-F_1 \, r \cos 30° + F_2 \, r \cos 30° = 0$$
$$F_1 \, r \sin 30° + F_2 \, r \sin 30° - F_3 \, r = 0$$

($\sin 30° = \frac{1}{2}$) and solving,

$$F_1 = F_2 = F_3 = \frac{W_a}{3}$$

While the forces depend on position and the value of Wa, they are not dependent on either p_i or A_i. Since $F_i = A_i\, p_i$, then the pressure in each unit could be found by

$$p_1 = \frac{W_a}{3A_1}\ ;\ p_2 = \frac{W_a}{3A_2}\ ;\ p_3 = \frac{W_a}{3A_3}$$

and for equal areas, $A_i = A$, all the pressures would be the same, $p_i = \frac{W_a}{3A}$. However, if the areas were unequal, the pressures would also be unequal but the forces would remain the same.

These three axial supports are an example of "displacement" supports. If we increase W_a, the force increases while its position remains almost unchanged. The other type of support, "force" or "astatic", would maintain an approximately uniform force even while changing position. These displacements supports "define" the mirror position relative to the structure.

Question 1. What would happen if all three units were interconnected by fluid lines?

For example, connect #1 to #3 and #3 to #2 as shown in **Figure 5**.

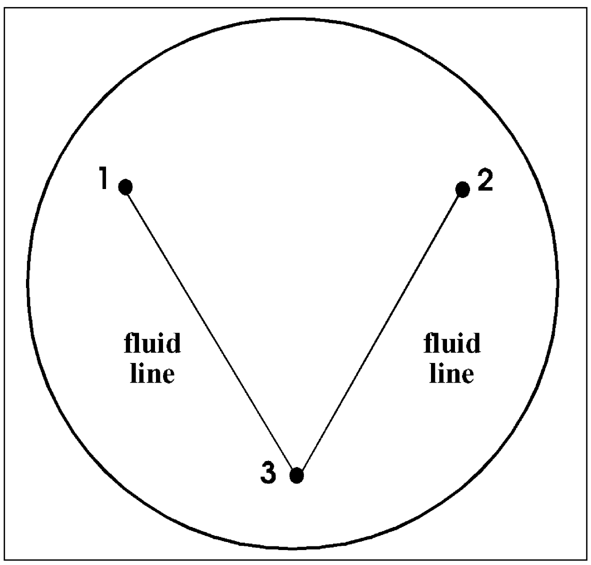

Figure 5.

Answer. The mirror would tilt uncontrollably. (Certainly bad for a mirror). Since the reason would not be obvious to everyone, some explanation follows. (Assume all assumptions above that led to constant pressures, $p = \frac{W_a}{3A}$.)

The roll diaphragms would allow the piston to move up or down with ideally no spring rate force so that fluid could be transferred from one unit to another, thereby causing the mirror to tilt. Why would the fluid transfer from one to another if the pressure is equal in each unit? The answer is that the hydrostatic head of fluid in the fluid lines causes a pressure difference. Assume the mirror has a positive rotation about the x axis. Then the vertical position of unit 3 is lower than units 1 and 2 by an amount $\frac{3}{2}$ r sin q. Then

$$p_3 = p_1 + \gamma\ r \sin\theta$$
$$p_2 = p_1$$

The force F_3 is now $> \frac{W_a}{3A}$, and the mirror is no longer in static equilibrium and will move up at point 3. Note that even though the pressure is greater in 3 than 1 or 2, the fluid does not move in that direction because the static head of fluid would have to be overcome. That is, the fluid moves from point 3 to point 1 or 2 only if $p_3 > p_1 + \gamma \frac{3}{2} r \sin\theta$.

If we found a way to require that $p_3 = p_1 + \gamma \frac{3}{2} r \sin\theta$ for equilibrium (and this could be accomplished in more than one way), would we want this interconnected system? *NO!* There is no spring rate in the three units when the pistons can move up or down, and any external load or change in the system would cause mirror tilts (undefined mirror).

Note: The following questions pertain to the original independent (non-interconnected) units.

Question 2. What happens if there is a leak in the bellowfram unit?

Answer. Then the mirror tilts until it comes to rest on the mechanical unit/cell. Small leaks can be overcome by building in a positive supply that can force fluid into or let fluid out of the system in response to a mirror tilt. A supply is needed for each of the three units. These supplies can also be used to adjust the mirror position whenever needed.

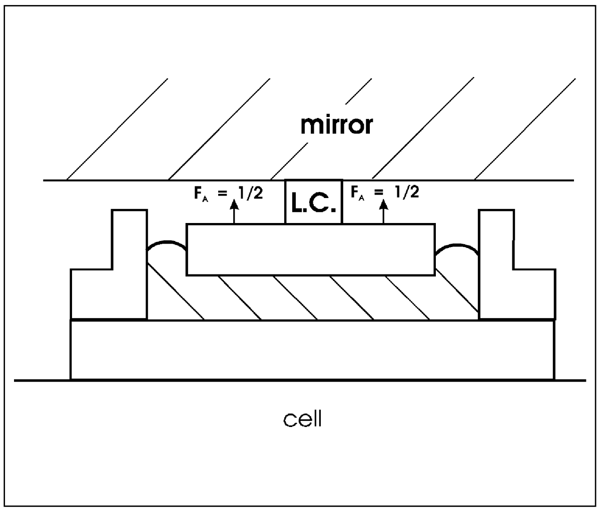

Figure 6.

Question 3. What happens if a force is applied to the piston of one unit?

For example, put a 1 unit load at number 3 (see **Figure 6**).

Answer. The L.C. reading does not change. The mirror does not know that an "active" force was applied. However, the pressure p_3 is now reduced to

$$p_3 = \frac{\frac{W_a}{3} - F_A}{A_3}$$

The combined force of F_A and the pressure is $\frac{W_a}{3}$ for equilibrium.

There is also a small tilt of the mirror. The sealed unit has a high spring rate, but the pressure change would mean that the piston would move up slightly in response to the upward force F_A.

Question 4. How does the system respond to forces that are applied to the mirror (external forces)?

For example, apply an axial load, F_E, at the coordinates ($\frac{-r}{\cos 30}$, 0) as shown in **Figure 7**.

Answer. From statics we can find the changes, DF_i, at each unit (i.e., $F_i = \frac{W_a}{3A_i} + DF_i$). This uses the same equilibrium relations as before. $SF = 0$; $SM_x = 0$; $SM_y = 0$. Setting up the equations and solving leads to

$$DF_1 = -1;\ DF_2 = \frac{1}{3};\ DF_3 = -\frac{1}{3}$$

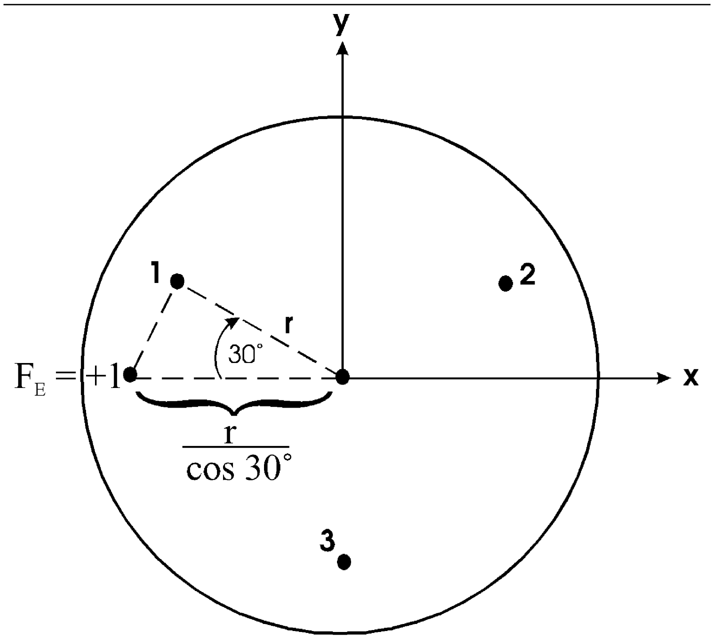

Figure 7.

This set of forces (the external load and the change in support forces which will be noted by the load cells) will cause two changes to the mirror:

1. The mirror surface will distort.
2. The mirror will tilt and/or piston.

The former will depend only on the mirror stiffness, while the latter will depend only on the combined stiffness of the hydraulic pads and the cell stiffness.

Question 5. What happens to the mirror if the cell warps (changes position in a differential way between the points 1, 2 or 3)?

Answer. The mirror will tilt/piston accordingly, but there would not be any distortion of the mirror surface.

Summary of Case 1

Using just three supports for 20% of the mirror weight and all axial external loads would not really be considered due to the large mirror distortion that would surely result. However we can consider these useful ideas:

1. The mirror would have a defined position (subject to the actual spring rate of the devices).
2. Several basic ideas were presented that will be the basis of other more complex systems.

Case 2. 6-point support @ 60° intervals on a common radius r_1.

If the six units were located at a radius of $.681r_o$ instead of three (see reference 1), then the δrms distortion for W_a would have far less distortion of the mirror surface, an improvement of about 8x over the 3-point result. Assume that we want six hydraulic support units in the configuration as shown in **Figure 8**. Use the same general assumptions as for the units in Case 1.

Question 1. What would be expected if 6 independent units were installed for the axial supports?

Answer. We no longer have a "kinematic" (3-point) support, and if the cell deflects differentially, then the load carried by each unit will vary, and there will be distortions of the mirror surface. This is a "displacement" support where the displacements and spring rates control the forces. The mirror would be "defined" but with the possibility of a poor figure. There is a possible solution for the non-uniform forces and that would be as follows. Assuming that there is a fluid supply for each unit, the amount of fluid could be metered in and out of the unit to maintain for force output (as measured on the load cell) to the desired value (here assumed to be equal forces on all six units, but it could also be controlled to maintain some other set of forces if desired).

Question 2. What would happened if the units were interconnected in pairs as shown below (Figure 9)?

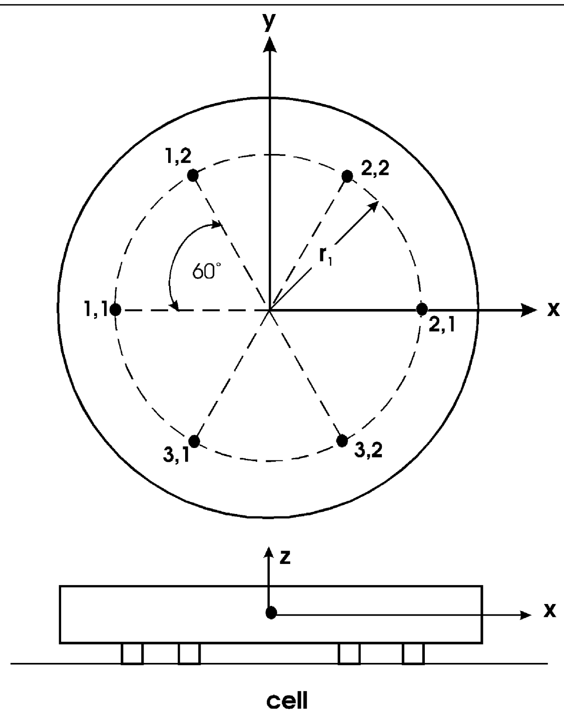

Figure 8.

The units are designated as i, j where i = 1, 2, 3 corresponding to the three units of Case 1, and j = 1, 2 for the individuals of a pair. Again, assume a positive zenith angle rotation about the x axis of θ.

Answer. There are very few drawbacks to this system. In fact, the only disadvantage is a distortion of the mirror surface since the forces at 1,1 and 2,1 are higher than those at the others because of the hydrostatic head between those units (see Case 1). The good points are:

1. Cell distortions are accommodated by transfer of fluid between the pairs. There will be some small tilt of the mirror if the sum of the displacements of each pair are different, but

[1] Nelson, et al., Telescope Mirror Supports, Plate Deflection on Point Supports, Report 74R, 1982.

this will be less, in general, than that of a three-point-support due to averaging over each pair.

2. The head pressure does not cause an uncontrollable tilt of the mirror since the units of higher pressure are located at y = 0.

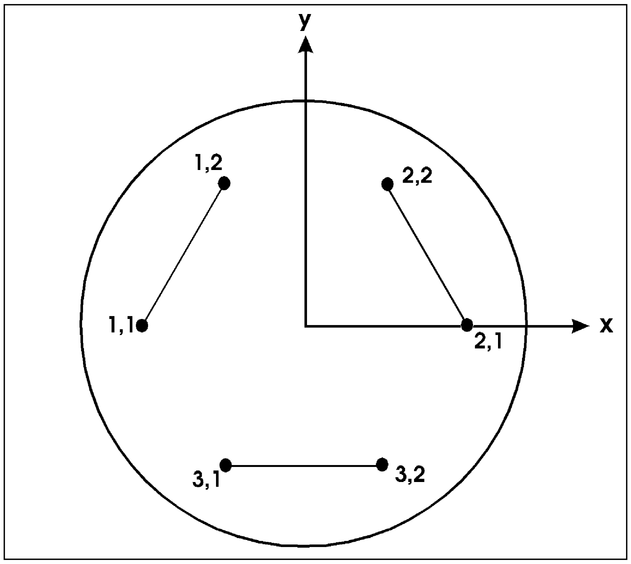

Figure 9.

3. The mirror is stable (with regard to rigid body motions) and is thus "defined". Each interconnected pair can rotate about the midpoint of the pair by transferring fluid from one unit to the other, but the midpoint does not move in space. These three midpoints play the same role as the three independent pads in Case 1.

Note: It should be noted that the distortion of the mirror surface due to the increased load at 1,1 an 2,1 may in fact be beneficial. Some mirrors (particularly concave mirrors) have a distortion due to the lateral supports that produces a high at the top and bottom, and a low at 1,1 and 2,1. Thus the effect of the increased force there can help compensate this lateral distortion; both effects are proportional to sin θ. One can set up the static equations and find that the forces are:

$$F_{11} = F_{21} = \frac{W}{6} \cos\theta + \gamma A r_1 \cos 30° \sin\theta$$

$$F_{12} = F_{22} = F_{31} = F_{32} = \frac{W}{6} \cos\theta - \frac{1}{3} \gamma A r_1 \cos 30° \sin\theta$$

Question 3. If the increased force at 1,1 is objectionable, it is possible to passively compensate the system to make all the forces equal?

Answer. Yes. (see Figure 10) A back-to-back unit can be installed with a stiff bracket to transfer the load of the compensation unit to the mirror.

All the support interconnection lines are duplicated by the compensation interconnection lines. These compensation lines/units can be vented to the atmosphere (zero bias) or not for some level of bias. While for this case it would be possible to install just the double ("compensated") units at 1,1 and 2,1, and a hydraulic line of the same length as the interconnection lines to the 1,2 and 2,2 positions, in general there would be a "compensated" unit at each position of a more complex system (or for an equatorial mount).

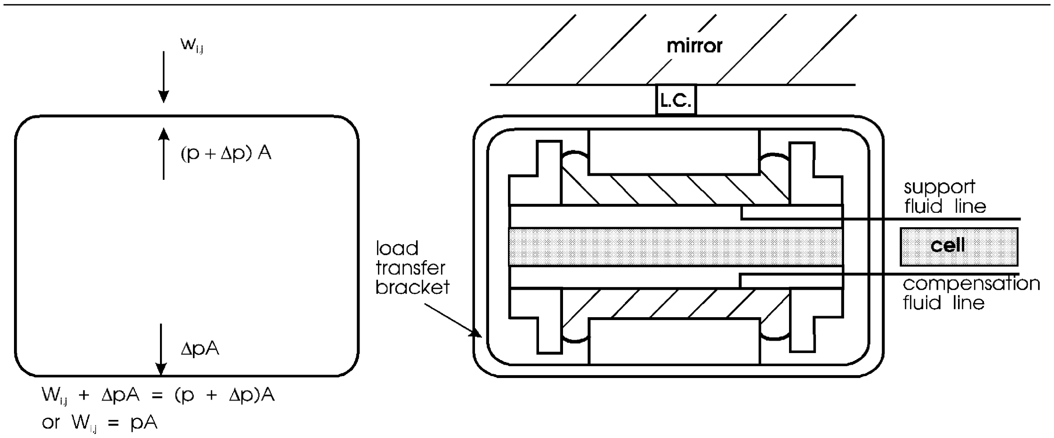

Figure 10.

Question 4. Can active forces be applied to the hydraulic units?

Answer. Yes, and one possibility is sketched below for each type of unit (**Figure 11**). The active force (a force under our control) is indicated as F_A and is applied to the load transfer bracket.

Question 5. If a unit active force is applied to a hydraulic unit, is this force then registered on the L.C. as being applied to the mirror?

Answer. No. The added load must produce an effect that still satisfies all of the static equilibrium relations. This can be demonstrated by a couple of examples.

Example 1. Given that $F_{A11} = 1;\ F_{Aij} = 0;\ i, j\ \ 1,1$

What is ΔF_{ij}, the changes in the forces as measured by the L.C.'s to the mirror?

The change at any pad unit is

$$\Delta F_{ij} = F_{Aij} + \Delta p_{ij} A_{ij}$$

where Δp_{ij} is the change in pressure in the support unit due to the application of an active force. For convenience we can assume that the areas, A_{ij}, are the same for all units and

$$\Delta p_{ij}\ A_{ij}\ \ F_{pij}$$

For the interconnected pairs, we know

$$F_{p11} = F_{p12};\ F_{p21} = F_{p22};\ F_{p31} = F_{p32}$$

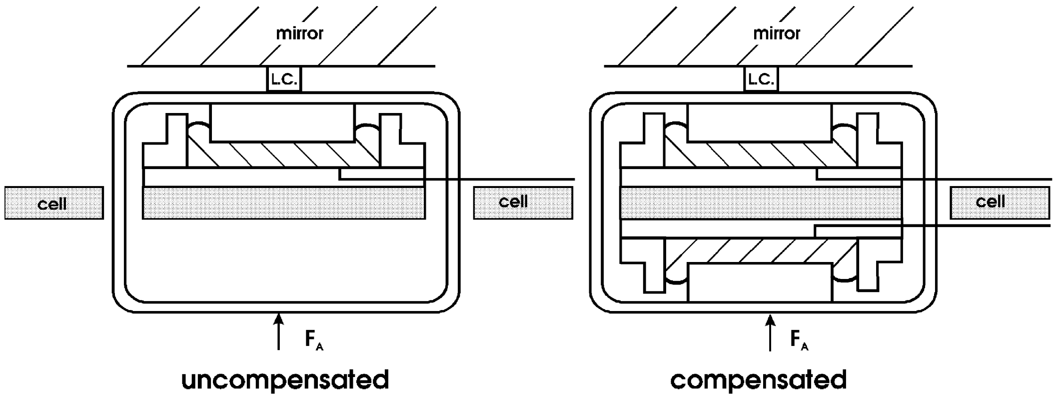

Figure 11.

since the pressure will be the same at each pad of the pair. We also have the equations of static equilibrium

$$\Sigma \Delta F_{ij} = 0; \quad \Sigma \Delta F_{ij} x_{ij} = 0; \quad \Sigma \Delta F_{ij} y_{ij} = 0$$

after writing the relations and solving we find that:

$$F_{p11} = F_{p12} = -\frac{1}{2}; \quad F_{p21} = F_{p22} = \frac{1}{6}; \quad F_{p31} = F_{p32} = -\frac{1}{6}$$

and therefore,

$$\Delta F_{11} = 1 - \frac{1}{2} = \frac{1}{2}; \quad \Delta F_{12} = 0 - \frac{1}{2} = -\frac{1}{2}$$

$$\Delta F_{21} = \Delta F_{22} = \frac{1}{6}; \quad \Delta F_{31} = \Delta F_{32} = -\frac{1}{6}$$

The forces on the mirror are indicated below (Figure 12) for $F_{A11} = 1$; $F_{Aij} = 0$; $i,j \neq 1,1$

Thus, while there is a deterministic relation between the active forces and the mirror forces, it is not a 1:1 transfer function.

What would be the forces on the mirror if $F_{A11} = F_{A12} = 1$; $F_{Aij} = 0$ otherwise?

This system could be solved (from "scratch") as was the previous example. We can also find the solution in other ways from previous information.

Solution 1. Use superposition and symmetry arguments (Figure 13).

Then by adding $[\Delta F_{ij}]_{11} + [\Delta F_{ij}]_{12}$ (Figure 13a).

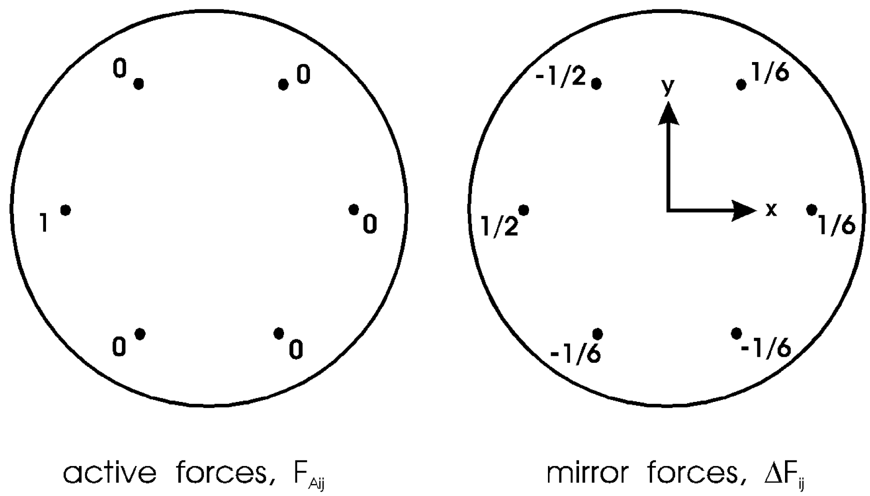

Figure 12.

The result of adding 1 unit force from the active control at 1,1 and 1,2 produces no mirror forces at all. While the first example produces mirror surface distortion, the second does not.

Solution 2. If we return to the 3-point case of independent hydraulic units (answer to question 3), it was noted that applying an active force to the unit did not change the force on the mirror, just the pressure in the unit. Here is the same situation. The 1 unit forces reduce the pressure in the pair, but do not affect the system otherwise.

Question 6. If an external load of 1 unit is applied directly to the upper mirror surface at the x,y position corresponding to unit 1,1, what will be the effect on the mirror and support system, assuming that all active forces are zero?

The external load on the mirror surface at x,y of $(-r_1, 0)$ is $F_E = -1$ (**Figure 14**).

Since all $F_{Aij} = 0$, then $\Delta F_{ij} = F_{pij}$

After setting up the static relations and solving, the result is (Figure 15).

The loads $^{D}F_{ij}$ *__on the mirror would cause distortion of the surface and some small tilt.__*

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Answer. We can add the two systems and find that the L.C.'s read as follows:

All L.C.'s are zero except for $^{D}F_{11} = 1$ @ location 1,1 (☞♓♑◆❒♏). 📂⌛

Figure 13.

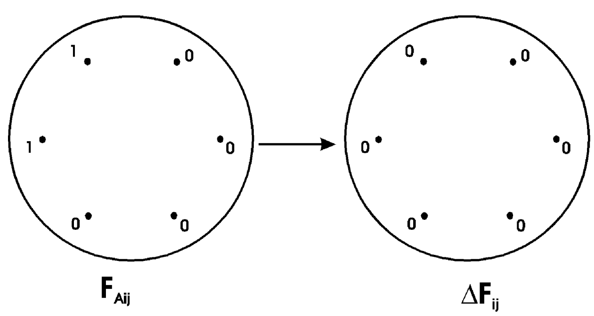

Figure 13a.

The load cell would be in compression and read 1 unit. The mirror would also be in compression, and there would be some small distortion at point 1,1 — otherwise, none.

Summary of Case 2

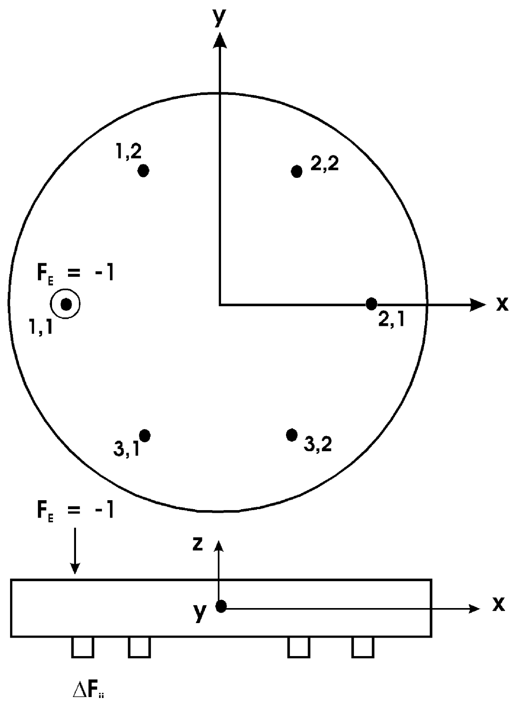

Figure 14.

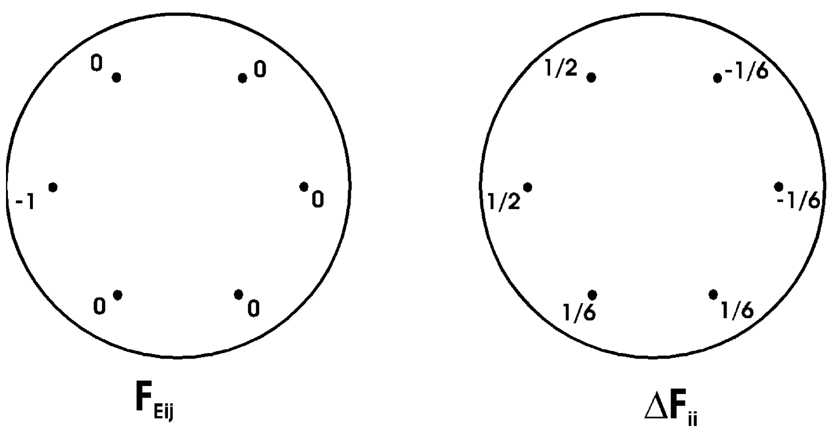

Figure 15.

The three pairs of interconnected units tend to act much like the three independent units. The mirror is defined even though there is some distortion when the hydraulic head is not compensated.

The compensation units are essentially the same hardware as the support units. This adds some cost to the system, but the ability to maintain forces for any zenith angle can have many advantages also.

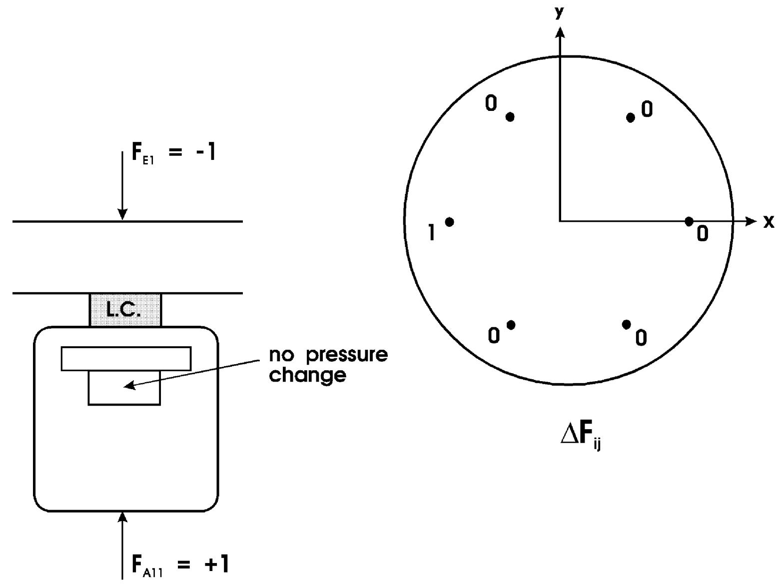

Figure 16.

Case 3. General ease of three zones of interconnected hydraulic units; n supports in each zone

One can continue to reduce the mirror surface distortion by increasing the number of support units in each zone. For the 8.2m mirror, it has been proposed that each of the three zones would have about 40 supports (120 total for the mirror). For the previous cases of n = 1 and n = 2, the relations were easily resolved, but the idea of n = 40 would suggest more formalism to avoid errors in the predicted forces. For this case the ground rules are:

1. There should be three independent zones of pressure supports. This will insure definition of the mirror.

2. All pressure supports in any one zone are interconnected. This will avoid overconstraints (cell deformation print through).

3. Hydraulic head pressure compensated units are used so that the pressure variation in a zone does not vary the force output.

For Case 1 of three points (n = 1) the three static equilibrium relations could have been written in matrix form as:

$$\begin{bmatrix} 1 & 1 & 1 \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{bmatrix} \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} = \begin{Bmatrix} W_a \\ 0 \\ 0 \end{Bmatrix}$$

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$$B = \begin{bmatrix} 1 & 1 & 1 \\ -r\cos 30 & r\cos 30 & 0 \\ .5r & .5r & -r \end{bmatrix}$$

while the force vector F = $\{F_1, F_2, F_3\}$ and the weight vector W = $\{W_a, 0, 0\}$. Solving the relation for F is

$$F = B^{-1}W$$

B should then be inverted, and the result can be found to be:

$$B^{-1} = \begin{bmatrix} 1/3 & -\frac{1}{2r\cos 30} & \frac{1}{3r} \\ 1/3 & \frac{1}{2r\cos 30} & \frac{1}{3r} \\ 1/3 & 0 & -\frac{2}{3r} \end{bmatrix}$$

and therefore,

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} = B^{-1} \begin{Bmatrix} W_a \\ 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} \frac{W_a}{3} \\ \frac{W_a}{3} \\ \frac{W_a}{3} \end{Bmatrix}$$

These are the load cell readings for just the weight component, W_a

For the case of the external load of F_E **$= +1$ at $x_E = -\frac{r}{\cos 30}$, $y_E = 0$, the three equilibrium** equations for the mirror are

$$B F + F_E = W$$

where $F_E = \{F_E, F_E x_E, F_E y_E\}$

Then, solving for F gives,

$$F = B^{-1}\{W - F_E\}$$

Therefore,

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} = B^{-1} \begin{Bmatrix} W_a - 1 \\ \frac{r}{\cos 30} \\ 0 \end{Bmatrix} = \begin{Bmatrix} \frac{W_a}{3} - \frac{1}{3} - \frac{1}{2\cos^2 30} \\ \frac{W_a}{3} - \frac{1}{3} + \frac{1}{2\cos^2 30} \\ \frac{W_a}{3} - \frac{1}{3} \end{Bmatrix}$$

However, $\cos^2 30° = 3/4$, and the vector then is $\{\frac{W_a}{3} - 1, \frac{W_a}{3} + \frac{1}{3}, \frac{W_a}{3} - \frac{1}{3}\}$

The change in force at the L.C.'s are then

$$\Delta F_i = \{-1, +1/3, -1/3\}$$

the same result as before.

Next, the Case 2 of six supports. The six supports are at radius r_1. The three equilibrium equations were:

$$F_{11} + F_{12} + F_{21} + F_{22} + F_{31} + F_{32} = W_a$$
$$F_{11}x_{11} + F_{12}x_{12} + F_{21}x_{21} + F_{22}x_{22} + F_{31}x_{31} + F_{32}x_{32} = 0$$
$$F_{11}y_{11} + F_{12}y_{12} + F_{21}y_{21} + F_{22}y_{22} + F_{31}y_{31} + F_{32}y_{32} = 0$$

Since there are six forces, F_{ij}, and only three equations, we could not solve directly for F_{ij} without some additional knowledge. For the examples in Case 2, it was assumed that

$$F_{11} = F_{12};\ F_{21} = F_{22};\ F_{31} = F_{32}$$

and with these relations, all the forces were found. We would like to allow for the possibility that not all forces in a zone are equal. We can assume that some relation exists between the forces in each zone and define

$$F_1^* \equiv F_{11} + F_{12};\ F_2^* = F_{21} + F_{22};\ F_3^* = F_{31} + F_{32}$$

ανδ

$$\bar{x}_1 = \frac{F_{11}x_{11} + F_{12}}{F_1^*}, \text{ εταχ.}$$

$$\bar{y}_1 = \frac{\Psi_{11}\Psi_{11} + F_{12}y}{F_1^*}, \text{ etc.}$$

Then the equations are:

$$F_1^* + F_2^* + F_3^* = W_a$$

$$\bar{x}_1 F_1^* + \bar{x}_2 F_2^* + \bar{x}_3 F_3^* =$$

$$\bar{y}_1 F_1^* + \bar{y}_2 F_2^* + \bar{\psi}_3 F_3^* =$$

or

$$B\,F^* = W$$

where B is

$$B = \begin{bmatrix} 1 & 1 & 1 \\ \bar{\xi} & \bar{x}_2 & \bar{x}_3 \\ \bar{y}_1 & \bar{y}_2 & \bar{y}_3 \end{bmatrix}$$

the same as before with x replaced by $\bar{x}_1$, etc. This reduces the equations to three, but one would still have to know $\bar{x}_i$ and $\bar{y}_i$ to find a solution. Assuming that the force is proportional to some constant pressure in the whole zone then

$$\Phi_{ij} = p A$$

where A_{ij} is the control (effective) area at that support and is known. Then

$$x_1 = \frac{\pi A_{11} x_{11} + p A_{12} x_{12}}{p A_{11} + \pi A_{12}} = \frac{A_{11} x_{11}}{A_{11}}$$

With the position and areas known, then all the $\bar{x}_i$ and $\bar{y}_i$ *values can be found.*

Therefore,

$$F^* = B^{-1} W$$

and when an external load is given

$$F^* = B^{-1}\{W - F_E\}$$

If we want just the effect of F_E, then

$$\Delta F_E^* = B^{-1}\{-F_E\}$$

[illegible] F_E [illegible]

$$\{F_E\} = \{-F_{Ex}, -F_{Ey}\} = \{+1, -r_1, 0\}$$

The areas in Case 2 were all equal and the values of $\bar{x}_i, \bar{y}_i$ are

$$\bar{x}_1 = -r\cos 30, \bar{x}_2 = r\cos 30; \bar{x}_3 =$$

$$\bar{y}_1 = r\sin 30, \bar{y}_2 = r\sin 30; \bar{y}_3 = -r$$

and $r = r_1 \cos 30°$. *(Note: this is not the same r as the Case 1 unless the three supports were located with this radius value). The result is*

$$\Delta F^*_E = \{+1, -1/3, +1/3\}$$

We then know that

$$\Delta F_{E11} + \Delta F_{E12} = 1$$

etc. For equal area (and constant pressure) the forces are

$$\Delta F_{E11} = \Delta F_{E12} = 1/2$$
$$\Delta F_{E21} = \Delta F_{E22} = -1/6$$
$$\Delta F_{E31} + \Delta F_{E32} = +1/6$$

Τηεσε αρε τηε χηανγεσ ιν τηε λοαδ χελλ ρεαδινγσ δυε το Φ_E and check with the previous results.

Question 1. What effect does an active force have when applied at one of the support units?

The active forces are not applied to the mirror directly. If they were, you could calculate their effect by calling them an external force and use the same relations above. It is almost the same except that the active force is applied through the load cell, and only at these specific locations. For a single active force at one actuator a,b (e.g., a = 1, b = 1 in the example of Case 2), the load cell reading will differ from the external force value only at the unit where it is applied and

$$\Delta F_{ab} = \Delta\Phi_{Eab} + \Phi_{\text{ab}}$$

while

$$\Delta\Phi_{ij} = \Delta F_{Eij} \text{ φορ } i,j \neq a b$$

We can illustrate this with the example of Case 2 where a,b = 1,1 and a + 1 force was applied. This is the same position as the external force above but of opposite sign. For this case, the result for an external load would be

$$\Delta F^*_E = \{-1, +1/3, -1/3\}$$

and

$$\Delta F_{E11} = \Delta F_{E12} = -1/2$$
$$\Delta F_{E21} = \Delta F_{E22}$$
$$\Delta F_{E31} = \Delta F_{E32} = -1/6$$

However,

$$\Delta F_{11} = \Delta F_{E11} + F_{A11} = -1/2 + 1 = 1/2$$

while the others are those values given above.

Now, instead of looking at n = 3,4. . . etc. we can just take the general case. Zones 1, 2 and 3 (Figure 17) are each assumed to have "n" supports, and each support is interconnected and compensated for hydraulic pressure. All the area are equal.

For each zone, i, (i = 1, 2, 3), we can define

$$F_i^* \equiv \sum_{j=1}^{n} F_{ij} \qquad \bar{x}_i = \frac{\sum_{j=1}^{n} F_{ij} x_{ij}}{F_i^*} ; \qquad \bar{y}_i = \frac{\sum_{j=1}^{n} F_{ij} y_{ij}}{F_i^*}$$

Then for static equilibrium with no external loads on the mirror or active forces, the equations are:

$$F^* = B^{-1} W; \qquad B = \begin{bmatrix} 1 & 1 & 1 \\ \bar{x}_1 & \bar{x}_2 & \bar{x}_3 \\ \bar{y}_1 & \bar{y}_2 & \bar{y}_3 \end{bmatrix}$$

As before, the values of $\bar{x}_i$ and $\bar{y}_i$ are

$$A_i^* = \sum_{j=1}^{n} A_{ij} ; \qquad \bar{x}_i = \frac{\sum_{j=1}^{n} A_{ij} x_{ij}}{A_i^*} ; \qquad \bar{y}_i = \frac{\sum_{j=1}^{n} A_{ij} y_{ij}}{A_i^*}$$

The change in forces ΔF^*_E *due to an external load is*

$$\Delta F^*_E = B^{-1}\{-F_E\}$$

where $\{-F_E\} = \{-F_E, -F_E x_E, -F_E y_E\}$. For an active force at a,b , there is both an effect as an external force and as an additional force at the support unit in question. The individual force change at some unit i, j is given by

$$\Delta F_{Eij} = \frac{A_{ij}}{A_i^*} \Delta F^*_{Ei}$$

At the unit a,b where there is an active force

$$\Delta F_{ab} = \Delta F_{Eab} + F_{Aab}$$

where ΔF_{Eab} is the effect of F_A **as an external load. As a final note, the change in load cell reading** ΔLC_{ab} ***is***

$$\Delta LC_{ab} = - \Delta\Phi_{ab}$$

since a positive force (upwards) will cause a compression (negative) of the load cell.

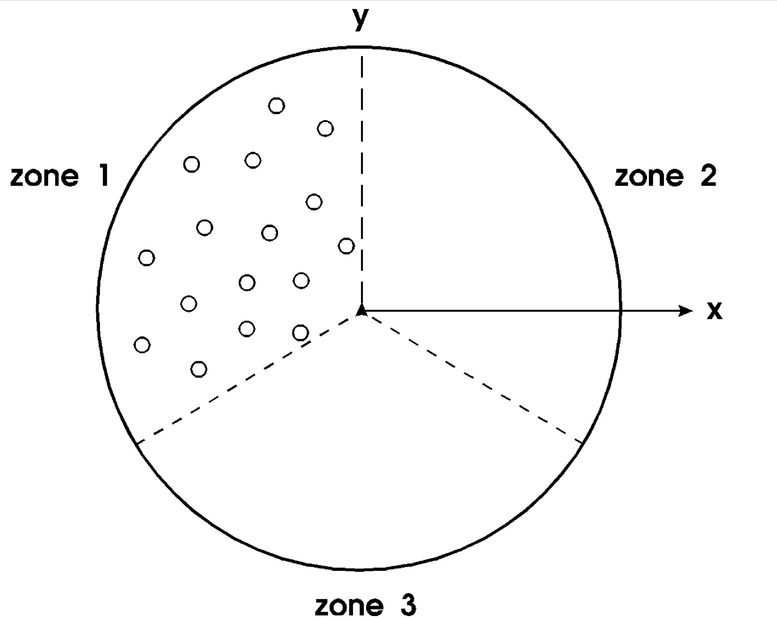

Figure 17.

Summary of Case 3

(in supports in three independent zones)

The case of n supports per zone is reduced to the same matrix equation as a three-point support by substituting the centroid $\bar{x}_i$ for ξ_i and $\bar{y}_i$ for γ. The inversion of this 3x3 matrix is known analytically and is known to be non-singular for all "practical" cases (e.g., each zone is approximately 120° of the surface area). The values of $\bar{x}_i$ *and* γ are determined from the areas and positions of the support units.

With the known value of B the sum of any known external or active forces can be related directly to the pressure in the three zones. These distributed forces can be related directly to the pressures in the three zones. These distributed forces can then be proportional to each support unit. The active forces add (at the unit where applied) to their effects as an external force.

Multiple Active/External Forces

The previous cases n = 2, 3, . . . specified the effect of a single external load or active force. In practice, there will be many such forces applied simultaneously. One technique to deal with many forces has already been mentioned: superposition of individual forces cases. The superposition of linear equations can also be expressed as

$$\begin{Bmatrix} \Delta F^*_{E1} \\ \Delta F^*_{E2} \\ \Delta F^*_{E3} \end{Bmatrix} = -B^{-1} \begin{Bmatrix} \sum_{m=1}^{mt} F_{Em} \\ \sum_{m=1}^{mt} F_{Em}\, x_{Em} \\ \sum_{m=1}^{mt} F_{Em}\, y_{Em} \end{Bmatrix}$$

where mt is the number of external forces.

We can define

$$F^*_E \equiv \sum_{m=1}^{mt} F_{Em}$$

$$-M^*_y \equiv \sum_{m=1}^{mt} F_{Em}\, x_{Em}$$

$$M^*_x \equiv \sum_{m=1}^{mt} F_{Em}\, y_{Em}$$

as the sum of external forces, the resultant external mome[illegible]
resultant external moment about the x axis, respectively.

Question 1. What is the result on the systems when multiple external forces are introduced simultaneously so that

$$F^*_E = 0,\quad M^*_y = 0,\quad M^*_x = 0$$

that is, whatever forces are applied have a net axial force of zero and zero moments about x and y?

Answer. We can see ΔF^*_{Ei} is zero which indicates that there is no change in pressure of any zone. The load cells would not change the reading.

Question 2. What is the result on the system when a multiple set of active forces is applied with the above requirements (i.e., sum of forces and moments are zero):

Considered as external forces, the result is the same as above (i.e., $\Delta F^*_{Ei} = 0$), but the load cells read values of

$$D_{ij} = {}^{D}F_{Eij} \quad {}_{Aij}$$

and since ${}^{D}F_{Eij}$ ... ${}_{Aij}$ as the individual active forces (but with the opposite sign). This almost sounds like a trivial result but, in fact, it is the only set of "pure" active forces that produce this result. As will be seen later, all active force systems devised to distort the mirror surface will have this zero sum and moments.

Question 3. What happens if all the active forces are equal over a whole zone?

Answer. *This is the extension on the previous examples of 3- and 6-point support. The pressure changes for that zone, but the load cells do not change, and no force is changed on the mirror. As an example, let the n active forces in zone 1 all be +1 unit and assume that all the areas are equal and the units are symetric in the zone. Then,*

$$F^*_E = n$$
$$-M^*_y = -nr\cos 30°$$
$$M = n\frac{r}{2}$$

Then

$$\begin{Bmatrix} DF_1^* \\ DF_2^* \\ DF \end{Bmatrix} = \begin{bmatrix} & \frac{-1}{2r\cos} & \frac{1}{3r} \\ 1/3 & \frac{1}{2\rho\cos 30} & \frac{1}{3r} \\ 1/3 & 0 & -\frac{2}{3r} \end{bmatrix} \begin{Bmatrix} n \\ -nr\cos 30 \\ \frac{nr}{2} \end{Bmatrix} = -\begin{Bmatrix} n \\ 0 \\ 0 \end{Bmatrix}$$

Therefore, in zone 1, each ${}^{D}F_{Eij} = -\frac{n}{n} = -1$

$$^{D}\Phi_{1j} = {}^{D}F_{E1\varphi} \; F_{A1\varphi} = -1 + 1 = 0$$

and in the other zone $\Delta F_{E2} = {}^{D}F_{E3} = 0$ and $F_{Aij} = 0$

Ωε χαν συμμαριζε βψ σαψινγ τηατ τηερε ισ α προχεδυρε το δετερμινε τηε χηανγε ιν Λ.Χ. ρεαδινγσ, βυτ τηε ρεσυλτ ωουλδ νοτ αλωαψσ βε ιντυιτιϖελψ οβϖιουσ. Σομε σπεχιαλ χασεσ ψιελδ τηατ αρε βοτη εασψ το ρεμεμβερ ανδ πραχτιχαλ.

Distributed External Forces

Τηε χασε οφ εξτερναλ λοαδσ ισ ειτηερ εασιερ ορ μορε διφφιχυλτ δεπενδινγ ον ονε϶σ ποιντ οφ ϖ Τηε αχτιϖε φορχεσ ωερε αππλιεδ τηρουγη τηε συππορτ υνιτσ (τηεψ ωουλδ νοτ ηαϖε το ηαϖε απτλιεδ ιν τηισ μαννερ, βυτ τηεν τηε ρελατιον ωουλδ ηαϖε βεεν διφφερεντ). Τηυσ τηε αχτιϖε φο ηαϖε χοορδινατεσ ιν χομμον ωιτη τηε ηψδραυλιχ συππορτσ, ανδ τηειρ νυμβερ ισ λιμιτεδ το μαξιμυμ. Τηε εξτερναλ λοαδσ χαν βε απτλιεδ ανψωηερε ον τηε μιρρορ συρφαχε (ορ ον τηε βα συρφαχε ορ σιδεσ, ετχ.), ανδ ηερε ωε ωιλλ λιμιτ τηε δισχυσσιον το εξτερναλ φορχε χομπονεντσ ι

[illegible]

[illegible] y, t) where p [illegible] is a pressure with some spatial distribution x, y and some changing function of time, t. The continuous distribution does not correspond to the discrete point locations, but it can be approximated by some finite grid of points, which in the limit, would approach the continuous distribution. If we let each grid point represent an area of DxDy, then

$$F_E(x, y) = -p(x, y)\, DxDy$$

The negative sign is needed since a positive pressure will produce a negative force in our coordinate system. Then the forces could be defined as

$$F_E^* \equiv \sum_{\text{all points}} [-p(x, y)\,\Delta x\, Dy]$$

$$-M_y^* \equiv \sum_{\text{all points}} [-xp(x, y)\, Dx\, Dy]$$

etc.

Another representation is the integral of the pressure as

$$F_E^* \equiv \iint_{\text{area}} -p(x, y)\,dx\,dy$$

$$-M_y^* \equiv \iint_{\text{area}} -xp(x, y)\,dx\,dy$$

$$M_y^* \equiv \iint_{\text{area}} -yp(x, y)\,dx\,dy$$

Σινχε ουρ αρεα ισ χιρχυλαρ, τηε ξ, ψ ρεπρεσεντατιον ισ νοτ νεχεσσαριλψ τηε μοστ χονϖενιεντ. Υσινγ πολαρ χοορδινατεσ ωιτη

$$x = r\cos q$$
$$y = r\sin q$$
$$dA = r\,dr\,dq$$

τηεν,

$$F_E^* = -\int_{-p}^{p}\int_{0}^{D/2} p(r, q)\,r\,dr\,dq$$

$$-M_y = -\int_{-p}^{p}\int_{0}^{D/2} p(r, q)\,r^2\cos q\,dr\,dq$$

$$M_x = -\int_{-\pi}^{\pi}\int_{0}^{D/2} p(r,\theta)\, r^2 \sin\theta \, d\theta \, dr$$

only due to external and pressure forces, is then

$$\begin{Bmatrix} \Delta F_{E1}^* \\ \Delta F_{E2}^* \\ \Delta F_{E3}^* \end{Bmatrix} = -\underline{B}^{-1} \begin{Bmatrix} F_E^* \\ -M_y^* \\ M_x^* \end{Bmatrix}$$

There will be change in the pressure of each zone that is calculated from this relation. If one wanted to know the surface distortion, then the mirror would be analyzed with the pressure loads on the top surface and the 3n loads on the bottom surface. It should also be noted that in the absence of active forces, the LC's will all read the same for each zone of equal area supports. That is to say that the spatial distribution information has been "lost" except for its overall force and moment. Each load cell in a zone, i, will be

$$\Delta LC_{i,j} = -\frac{\Delta F_{E_i}^*}{A_i}$$

To analyze what the static pressure does to the system without active forces, we can assume the static pressure spatial distribution could be represented by a polynomial such as

$$p(\rho,\theta) \approx C_0 + C_1 r\cos\theta + C_2 r\sin\theta + C_3(2r^2-1) + C_4(r^2\cos 2\theta) + C_5(r^2\sin 2\theta)$$

ωηερερ= 2ρ/Δ. Τηισ ισ τηε Ζερνικε σεριεσ ωηερε ρ ισ τηε νορμαλιζεδ ραδιυσ φρομ 0 το 1. Νοω εαχη τερμ χαν βε αναλψζεδ σεπαρατελψ. Τηε ιντεγραλσ χαν βε χονϖερτεδ το

$$F_E^* = \frac{D^2}{4}\int_{-\pi}^{\pi}\int_{0}^{1} p(r,\theta)\, r \, dr \, d\theta$$

$$-M_y^* = -\frac{D^3}{8}\int_{-\pi}^{\pi}\int_{0}^{1} p(r,\theta)\, r^2 \sin\theta \, d\theta \, dr$$

$$M_x^* = -\frac{D^3}{8}\int_{-\pi}^{\pi}\int_{0}^{1} p(r,\theta)\, r^2 \sin\theta \, d\theta \, dr$$

For C_0, $F_E^* = C_0 \pi D^2/4$ while $M_y^* = M_x^* = 0$.

The vector $\left\{C_0 \frac{\pi D^2}{4}, 0, 0\right\}$ plays the same role as $\{W_a, 0, 0\}$, ανδ εαχη ζονε ηασ αν ινχρεασε ιν pressure of $\frac{C_0 \pi D^2}{12 N A}$. As long as $C_0 \pi D^2/4 << W_a$ ...

resisted without much mirror surface distortion. Because of the orthogonality of the Zer[nike] series, C_0 is also the only term that $F_E^* \neq 0$.

For $C_1 r\cos\theta$; $F_E^* = 0$; $-M_y^* = \frac{C_1 p D^3}{32}$; $M_x^* = 0$

There is an equal and opposite pressure in zone 1 and 2 and zero pressure in zone 3.

For $C_2 r\sin\theta$; $F_E^* = 0$; $-M_y^* = 0$; $M_x^* = \frac{C_2 p D^3}{32}$

Here pressure in zones 1 and 2 are equal to each other and are half the pressure of zone 3. Zone 3 is opposite in sign to 1 and 2.

For $C_3(2r^2 - 1)$ and all higher order terms; $F_E^* = -M_y^* = M_x^* = 0.$

There is thus no pressure in the zones due to these terms. When M_y^* or $M_x^* \neq 0$, the pressure in the hydraulic system resists the moment due to the wind, but the actual distribution of support pressures is not adequate to prevent distortion of the surface. When $M_y^* = M_x^* = 0$, there is no pressure induced in the supports. The mirror distorts much the same way as for the active force systems where $F^* = M_y^* = M_x^* = 0$ as noted earlier. The cell/supports do not resist in any way.

For the three non-zero terms $F_E^*(C_0)$, $M_y^*(C_1)$ and $M_x^*(C_2)$) above, the LC's will have a small reading in each zone and there will be some coupling to the cell. For all the rest of the Zernike terms there will not be any LC readings (for static pressure patterns)

The Role of Pressure Bias in the Head Compensation System

With reference to Figure 11, it is obvious that the pressure (positive pressure only) in the compensation unit produces a force that has the same effect as a negative value of F_A. Not all systems need hydraulic head compensation*, but those that do could produce the head correction with the active force system. The disadvantage of doing this is the added requirement to the active system and lack of a passive support mode for the mirror. One advantage might be that the added cost on the active system would be less than the cost of the compensation unit.

Once the compensation system has been included as part of the support system, there are several options to consider.

1. Each of the three zones has a compensation system with the same units and plumbing pattern as the support units. The upper unit (Alt-Az only) is vented to the atmosphere.
2. Each of the three zones is as in No. 1, but the units are sealed and a pressure (bias) can be added to each system.
3. All units of the compensation system are interconnected and vented to the atmosphere.
4. All units are interconnected as in No. 3, but pressurized with a bias.

Question 1. What advantages and disadvantages are there in using a bias?

Answer. The major advantage is the change in spring rate for the hydraulic support units. The spring rate of the collective set of a zone affects the definition of the mirror and the frequency response of the system. We would like the highest spring rate possible. Pressure bias in the compensation units adds to the pressure in the support unit. Once the bias is more than the equivalent of the axial load carried (~ W_a**), the spring rate of the total system will be about double that of the support units alone.**

The disadvantages are as follows. The support units must carry a pressure about double the unbiased value. Supplies must add fluid at this higher value. A small leak in an unbiased compensation system would not affect the system for a long time, but in a biased system it will be noticed almost immediately. To change the mirror position in piston requires that supplies of both systems be accomplished simultaneously.

We chose to use a bias to double the spring rate and will work around the practical problems.

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⌘□■♏⬧✍

Answer. Τηε σινγλε ζονε ρεθυιρεσ ονλψ ονε φλυιδ συππλψ ινστεαδ οφ τηρεε (φορ τηρεε ζονε Αλσο, ιφ α σινγλε χομπενσατιον ζονε ισ υσεδ, τηε διφφερεντιαλ πρεσυρε βετωεεν τηε ζονεσ ωιλλ χηανγε ωιτη ζενιτη ανγλε.

ΕΞΕΧΥΤΙςΕ ΣΥΜΜΑΡΨ

Τηε μενισχυσ μιρρορ ωιλλ βε συβφεχτεδ το μανψ φορχεσ, βοτη διστριβυτεδ ανδ χονχεντρατεδ. Τηε φιναλ φιγυρε τηατ τηε μιρρορ εξηιβιτσ ισ σενσιτιϖε το εϖεν σμαλλ ϖαριατιονσ ιν φορχε λεϖελ. Τηε αναλψσισ χαπαβιλιτψ εξιστσ το χαλχυλατε τηε μιρρο διστορτιον φορ α γιϖεν σετ οφ φορχεσ ανδ βουνδαρψ χονστραιντσ. Τηε προποσεδ ηψδραυλιχ διστριβυτεδ συππορτ σψστεμ ωιλλ ρεδιστριβυτε φορχεσ ιμποσεδ βψ εξτερναλ λοαδσ, τηε αχτιϖε φορχε σψστεμ, ετχ. ιν ωαψσ τηατ μιγητ νοτ αλωαψσ βε ιντυιτιϖελψ οβϖιουσ. Τηισ ρεπορτ εξπλαινσ τηε ωορκινγσ οφ διστριβυτεδ δεφινινγ σψστεμσ ανδ ινδιχατεσ ωαψσ οφ πρεδιχτινγ τηε ρεσυλταντ φορχε σψστεμ φορ στατιχ, εθυιλιβριυμ χονδιτιονσ.