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Subject: Image Smear Error Budget with Required Servo Bandwidth and Sampling Rate

Introduction

The value of an active optics system is that a more stable image may be obtained by tip-tilting the secondary mirror to remove random image motion. If the random motion of the image is relatively slow compared to the servo driving the secondary, then compensation will be good and the resulting image motion will be small.

Summary

The requirment for reducing image smear due to atmospheric effects may be met with a servo closed loop bandwidth of 15Hz sampled at 100Hz. The requirment for reducing image smear due to wind shake may be met with a servo closed loop bandwidth of 40Hz sampled at 200Hz.

Problem

The total image smear error budget is 0.03 arcsec. This represents an increase in the 50% encircled energy. For simulation purposes it is easier to consider the target to be a point and therefore the RMS motion of the image centroid is more relevant. It is believed that the relationship between 50% encircled energy and centroid motion is near 1:1.35, so the error budget for centroid motion is taken to be 0.022 arcsec RMS or approximately 0.108 micro-radians RMS per axis.

The 0.108 micro-radian error budget is for wind-shake and other unmodelled disturbance torques. These torques are considered to be statistically independent so the net RMS level is obtained by RSS-ing (root sum squaring) all of the components. The target for wind shake is that it take up one quarter of the total image smear budget power or one half of the RMS, so the target wind shake after compensation is 0.054 micro-radians RMS.

Atmospheric effects are considered separately from the image smear error budget with the requirement that the power in atmospheric noise be reduced by 90%. The uncompensated atmospheric image motion is calculated to be 0.96E-12 rad^2. Multiplying this by 0.1 and taking the square root gives the allowable compensated RMS of 0.31 micro-rad. After discussions with representatives of Starfire Labs, correlation noise will account for 0.083 micro-rad of the 0.31

micro-rad, which means that compensated atmospheric noise must be reduced to 0.3 micro-rad RMS.

The 0.08 micro-rad correlation noise is calculated assuming that

$$\text{Enoise} = \frac{\text{lambda}/R0}{SNR}$$

where lambda= 2.2 microns
R0 = 1.33 meters
SNR = 20.

Disturbance Torques

The table below shows the expected RMS level of disturbance induced image motion both before and after compensation. Note that the requirement on atmospheric induced error is met for a 15Hz servo bandwidth operating at 100Hz sampling rate but that the requirement on windshake mandates a much higher bandwidth of 40Hz and sampling rate of 200Hz.

Disturbance Torque	Uncompensated RMS (micro-radians)	Compensated with BW=15Hz,rate=100Hz (micro-radians)	Compensated with BW=40Hz,rate=200H (micro-radians)
Atmospheric	0.98	0.25	0.11
Wind shake (Tz)	0.85	0.23	0.10
Other unmodelled	0.17	0.06	0.04

Table 1: List of Disturbance Torques Before and After Compensation

The unmodelled disturbance torques include bearing friction, servo torque ripple and servo reaction torques transmitted through the telescope structure. The compensated values above assume that the disturbances have a bandwidth of 1Hz. If the actual bandwidth is less, then filtering action will improve and a larger uncompensated RMS level will be tolerable. The tradeoffs are shown below in the results section.

Figures 1,2 and 3 show the power spectra for atmospheric induced errors before compensation, after compensation with 15Hz bandwidth and after compensation with 40Hz bandwidth. The uncompensated power spectrum is a modified Greenwood distribution which falls off like the 2/3rds power of frequency up to a certain cutoff then falls off like the 11/3rds power afterwards. This distribution is possibly overly conservative, and more accurate data is expected from Starfire Labs soon. If the atmospheric data used for this report were overly conservative then the new data would require less bandwidth and slower sampling than the 15Hz and 100Hz quoted here.

The spectra for wind shake in the z-direction, Tz, before and after the same compensation are shown in figures 4, 5 and 6. The corresponding spectra for Tx are expected to be the same for an orthogonal wind. The example spectra for unmodelled disturbance torques are in figures 7, 8, and 9. The unmodelled disturbance torques are modeled as first order Markov processes, that is what noise through a low pass filter with a single pole. Results would improve slightly for higher order models of a given bandwidth and RMS because the fraction of power at higher frequencies would be smaller. All spectra are double-sided, meaning that it is necessary to integrate from negative infinity to positive infinity frequency to encompass all of the energy.

Tip/tilt Model

Figure 10 shows the block diagram representation of the single axis model of the secondary tip/tilt system used for this study. The input at the left, theta_c is assumed to be a random process with some known spectrum representing uncompensated image motion. The variable theta_o represents the actual position of the secondary and the difference between theta_c and theta_o is the error after compensation which is desired to be near zero. The error signal passes through a sample and hold to give the measured error theta_m. The sample and hold allows the analog model of the servo to interact with the digital control system and causes a small amount of degradation.

The measured error is fed to the servo model which acts to attempt to drive the error to zero. The servo model block is

$$\frac{\text{theta\_o(s)}}{\text{theta\_m(s)}} = \frac{\text{w0*w0}}{\text{s\^{}2 + s*2*w0}}$$

where w0=2*pi*BW with BW being the desired closed loop bandwidth. Good design dictates that sampling rate should be 5-20 times the closed loop bandwidth. It is noticed that for a bandwidth of 15 Hz and a sample rate of 100Hz the closed loop has a phase margin of 76 degrees. The signal processing delay of 500 microseconds has been neglected in the model because it is small. This delay gives 2.7 degrees of phase lag at 15Hz, which will put the phase margin at 73 degrees, still above our target of 70 degrees.

Figure 11 shows the squared magnitude of the filter function from theta_c to theta_e for a 15Hz bandwidth servo loop and 100Hz sampling. The compensated error spectrum (e.g. fig. 2) is found by multiplying this curve point by point times the input spectrum (e.g. fig. 1). The compensated error spectrum is then integrated over frequency to give compensated power and then taking the square root gives compensated RMS. Note that both uncompensated atmospheric (fig. 1) and uncompensated windshake (fig. 4) start to fall off around 2.5 Hz. At this frequency the squared filter function is around 0.1, which attenuates the input power by a factor of 10. Another interesting point to notice about figure 11 is the overshoot magnitude of 1.3 near 18Hz. The filter not only fails to attenuate input power at this frequency but actually amplifies it. The overshoot is undesirable but is an inevitable part of having 70 degrees of phase margin. Increasing the sampling rate for a given servo bandwidth will improve the phase margin and shrink the overshoot.

Results

The following table shows the resulting image motion after compensation for a range of servo bandwidths. The sampling rates for each bandwidth are also shown. The "compensated other disturbance" values shown are chosen such that for the z-axis the RSS of Tz and the "other" is equal to the total error budget of 0.2 micro-rad.

Servo BW (Hz)	Sample Rate (Hz)	Compensated Atmospheric (micro-rad)	Compensated Windshake Tz (micro-rad)	Compensated other distrb (micro-rad)
5	100	0.54	0.53	-----
7	100	0.44	0.43	-----
10	100	0.34	0.32	-----
15	100	0.25	0.23	-----
20	100	0.20	0.18	-----
25	200	0.16	0.15	-----
30	200	0.14	0.13	-----
35	200	0.12	0.11	-----
40	200	0.11	0.10	0.04

Table 2: RMS Image Motion After Compensation for Various Disturbance Torques

Table 3 below shows the effect of the disturbance toorque bandwidth on the servo loop's capability to reject it.

Servo BW (Hz)	Sample Rate (Hz)	Compensated other distrb (micro-rad)	Uncomp at 0.1 Hz BW (micro-rad)	Uncomp at 1.0 Hz BW (micro-rad)	Uncomp at 10.0 HzBW (micro-rad)
40	200	0.04	0.53	0.17	0.07

Table 3: Allowable Uncompensated Other Disturbance Torques

Note that as the bandwidth of the disturbance torque approaches the bandwidth of the servo loop, for example the 40 Hz servo and 10Hz disturbance, the servo loop is capable of removing only a small amount of the disturbance power and the compensated RMS (0.04 micro-rad) approaches the uncompensated RMS (0.07 micro-rad). Alternately, if the servo loop is very fast compared to the disturbance torque, then the uncompensated RMS (0.53) is attenuated greatly to give the compensated RMS (0.04). This is a savings of factor 13 in RMS or 178 in power. If the unmodelled disturbance torques have bandwidth in excess of 10Hz, there is very little savings to be had in compensation so they must be kept small.

Conclusion

Reduction of atmospheric induced image smear turns out to be an easier task than reduction of the windshake induced image smear. It is easier in the sense of requiring lower servo bandwidth and corresponding lower sampling rate: 15Hz bandwidth and 100Hz sampling vs. 40Hz bandwidth and 200Hz sampling. The reason for this difference is partly that windshake has proportionally more of its energy at higher frequencies and partly that the requirements are different. It is required to reduce atmospheric noise by 90% in power which corresponds to a factor of 0.31 in RMS. The windshake must be reduced from 0.85 microrad RMS to 0.1 microrad RMS, a factor of 0.12 in RMS.

List of Figures

Figure 11 Example Filter Attenuation for 15Hz Bandwidth