GEMINI 8-METRE TELESCOPES PROJECT
CONTROLS GROUP

To: File

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Subject: A Method for Determining Tip-Tilt Secondary Bandwidth and Power Requirements

Reference: [1]Bandwidth and Power Requirements for Tip-Tilt, R. McGonegal , January 17, 1993.
[2]Power Requirements for Chopper, R. McGonegal, January 17,1993.
[3]Tip-Tilt Chopper Control Study and Power Requirements, M. Burns March 22, 1993.

Problem

It is required to have a tip-tilt secondary mirror system which tracks out 90% or more of the power associated with atmospheric noise on a stars apparent motion. The resulting system must be tolerant of a reasonable number of sample delays and boxcar averaging steps and have acceptable gain and phase margins.

Summary

Two models of atmospheric noise are considered, the first being the Greenwood model of ref [1] and the second a modified version of the Greenwood model having significant energy at higher frequencies. For the former and more benign case, bandwidth of 5 Hz is sufficient to meet the noise rejection criterion. Achieving a phase margin of 70 degrees pushes the sampling rate requirement to 1Khz to be able to tolerate 5 delays and 5 boxcar averaging steps.

For the modified Greenwood noise model, a 15Hz bandwidth servo system is required to reject 90% of the noise. A sampling rate of 1Khz gives a phase margin of 66 deg for 3 delays and 3 boxcar averaging steps. A lower sampling rate would result if the phase margin requirement were relaxed, but this is not recommended since unmodelled delays and lags could greatly degrade performance by pushing the servo loop into a region of excessively light damping or even instability.

Both noise models require less than 1 micro-watt of actuator power. These simulation runs can be repeated when we settle on a believable model of the wind.

Noise Model: Greenwood

Figure 1 shows the power spectral density (in rad^2/Hz) of the Greenwood type noise process as represented in the reference [1]. This power spectrum can be represented in terms of the normalized frequency x=f/f0 as

$$H(x) = \frac{x^{-2/3}}{(2\pi)^2} \qquad \text{for } x < 0.332$$

$$H(x) = \frac{x^{-2/3} * (1.12-0.361 x)}{(2\pi)^2} \qquad \text{for } 0.332<x<3.1$$

$$= \text{ near zero} \qquad \text{for } 3.1< x \;,$$

where

$$f0= v/(\pi D) \;.$$

The un-normalized power spectrum , in rad^2/Hz is

$$F(f) = \frac{H(x)}{0.481 f0 (r0/D)^{5/3} (D/lam)^2}$$

where
- v = velocity of seeing layer = 20 m/s
- r0 = aperture = 1000 mm
- D = mirror diameter = 8000 mm
- lam = wavelength of interest =0.0022 mm .

Thus f0 is found to be around 0.8 Hz. The above spectral density, F(f) is referred to as the Greenwood spectral density in this report.

Noise Model: Modified Greenwood

A better approximation to the atmospheric noise is to let the tail at higher frequencies fall off like f^(-11/3) rather than being considered negligible above the cutoff frequency of 3.1*f0=2.5Hz. Though it still falls off quite quickly, it will be shown that the total energy in this tail is significant both because it extends over a broad frequency range and because it is that part of the spectrum over which we have poor disturbance rejection. The resulting spectrum is referred to as the Modified Greenwood in this report. Thus:

$$H(x) = \frac{x^{(-2/3)}}{(2\pi)^2} \qquad \text{for } x < 3.1$$

$$H(x) = \frac{c \, x^{(-11/3)}}{(2\pi)^2} \qquad \text{for } x > 3.1 \;.$$

The constant c is chosen such that the spectral density is continuous at x=3.1, thus c=3.1^(9/3)=29.8 .

The total normalized power in this can be easily be found by integrating H(x) over x :

$$P_{net} = \int_0^{3.1} \frac{x^{(-2/3)}}{(2\pi)^2}\,dx + \int_{3.1}^{\infty} \frac{29.8\, x^{(-11/3)}}{(2\pi)^2}\,dx$$

$$P_{net} = 0.111 + 0.014 = 0.125 \;,$$

so roughly one eighth of the total power is in the region above the "cutoff" frequency. This might seem like a relatively small amount, but it is important to remember that these higher frequencies are where filtering is weakest, so it will contribute strongly to the residual power after compensation.

Consider a simple estimate of bandwidth required to remove some chosen amount of the error power. Assume we have a best case filter with a normalized cutoff frequency xcut. For the purposes of this simple estimate, assume that it is perfectly high pass, that is, that it removes all of the energy below xcut and leaves untouched the energy above xcut. Then the power remaining after compensation will be:

$$P_{com} = \int_{xcut}^{\infty} \frac{29.8\, x^{(-11/3)}}{(2\pi)^2}\,dx = 0.283 \cdot xcut^{(-8/3)} \;.$$

The table below shows the normalized power and the fractional power which remains

xcut	Pcom	Pcom/Pnet
3.5	1.0E-2	8.0%
4	7.0E-3	5.6%
5	3.9E-3	3.1%
7	1.6E-3	1.3%
9	8.1E-4	0.6%

So it can be seen that if we wish to make the compensated power less than 10% of the total power, a normalized bandwidth of at least 3.5 (i.e. 2.8 Hz since f0=0.8Hz) is needed, and if we wish remove all but 1% of the total power, a bandwidth of about 8 (i.e. 6.5 Hz) is required. These numbers are useful in that they tell us where to start looking for filters. They represent the very best case, since the assumed filter has an infinitely sharp cutoff.

System Model

Figure 3 shows a simple block diagram representing the MatrixX model used in the simulations. The Greenwood type noise represents the apparent position of the star due to atmospheric effects. This angular position is called theta-command and is given the symbol theta_c here. The actual pointing of the mirror is labeled theta_o and the difference between the commanded and actual is the error denoted theta_e. This error is passed through Ndelay pure delays and Mbox steps of a boxcar averager to produce a measured position theta_m which drives the servo thus closing the servo loop. The measured position also drives the power calculation block, labeled Power in Figure 3.

It should be noted that since a time response is not desired all calculations in this simulation are performed in the frequency domain. Frequency domain calculations of noise power and servo power are done by using the relationship between input and output power spectral density :

Phi_out(w) = Phi_in(w) * |H(w)|^2 ,

where Phi_out(w) is the output power spectrum as a function of frequency, Phi_in(w) the input spectrum, and H(w) the transfer function from input to output. The total power, for example in the error signal, may thus be computed by integrating Phi_out(w) over frequency.

To evaluate the noise rejection it is necessary to compute the transfer function from theta_c to theta_e. Computation of servo power requires the transfer function from theta_c to I. Another transfer function which is of value in computing gain and phase margins is the forward path from theta_e to theta_o.

The transfer functions of the individual blocks N_delay and M_box are most naturally represented in the discrete, or z domain, while the servo and power are given in the continuous or s domain. The procedure used here is to transform the servo and power blocks to the z-domain, find the required input-output transfer functions in the z-domain, and get Bode plots for power integration and phase margin calculations. Note that one could also choose to transform the delay elements to the s-domain for example by way of a Pade approximation. This method was not chosen due to the notorious inaccuracies of representing a long delay by a relatively small number of poles and zeroes in the s-domain. The transformation of the servo to the discrete domain via Tustin's method is expected to have good fidelity since the sampling rate is much higher than the servo bandwidth (from 5 to 300 times for the candidate systems).

The Ndelay block was obtained simply by cascading Ndelay number of 1/z elements. The boxcar averager of order Mbox was obtained by adding the past Mbox number of measurements and dividing by Mbox. In the z-domain this can be represented:

$$\text{Hmbox}(z) = \left(1 + \frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3} \quad \ldots\ldots \quad \text{Mbox terms}\right) / \text{Mbox}$$

So for example a 2 step boxcar averager would be the present plus the last term divided by 2:

$$\text{Hmbox2}(z) = (1 + 1/z)/2 = (z + 1)/2z.$$

The continuous domain servo model transfer function (radians/radian) from error signal to mirror angle is:

$$\frac{\text{theta\_o}(s)}{\text{theta\_m}(s)} = \frac{w0^2}{s^2 + s*2*\text{zeta}*w0}$$

where w0 is 2pi times the bandwidth in Hertz and zeta is the damping coefficient. The damping coefficient was chosen conservatively at 1.0 with the expectation that adding delay would decrease damping by adding phase. Note that the above transfer function gives the more well known closed-loop transfer function from theta_c to theta_o :

$$\frac{\text{theta\_o}(s)}{\text{theta\_c}(s)} = \frac{1}{s^2 + s*2*\text{zeta}*w0 + w0^2}$$

when the unity gain loop is closed around it without any delay or boxcar averaging.

The following analysis shows how to get the transfer function from the measured angle theta_m to required current I. From references [2] and [3] the transfer function from torque to mirror angle is:

$$\frac{\text{theta\_o}(s)}{\text{torque}(s)} = \frac{1}{4*s^2 + 10*s + 6500} .$$

From [3] noting that torque is related to actuator current by the factor i2trq=15.8 A/Nm , the above equation yields :

$$\frac{\text{theta\_o}(s)}{I(s)} = \frac{15.8}{4*s^2 + 10*s + 6500} .$$

The last equation above may be divided by the servo model to get the relationship between theta_m and current I:

$$\frac{I(s)}{\text{theta\_m}(s)} = \frac{\text{theta\_o}(s)}{\text{theta\_m}(s)} \text{ divided by } \frac{\text{theta\_o}(s)}{I(s)} = \frac{wo^2(4*s^2 + 10*s + 6500)}{15.8(s^2 + s*2*zeta*w0)} .$$

This transfer function may be used to get the power spectral density of the current which can be integrated over frequency to give expected current squared. From ref [3] the current squared times 3 (to account for resistance=2 ohms and 3 actuators having unequal currents) will give net power required by the 3 actuators.

Simulation Results

Appendices A and B contain the results of a large number of batch runs for the Greenwood and Modified Greenwood models of atmospheric noise. Each appendix required approximately 54 hours to run on a 486 DX2-50MHz type machine, and spans 4 dimensions which are:

sample rate = delt = 0.01, 0.003, 0.001 sec
servo bandwidth = bw = 3, 5, 7, 10, 15, 20 Hz
number delays = ndelay = 1, 2, 3, 4, 5, 6
number boxcar averagers = mbox = 1, 2, 3, 4, 5, 6 .

The output pages occur in pairs for a given sampling rate-bandwidth combination, with the first page having outputs for

P_error = power remaining after compensation (rad^2)
P_servo = power required for 3 servo actuators (Watts)
P_ratio = P_error / Ptotal= fractional power remaining after compensation

and the second page having outputs corresponding to stability margins:

Gnmarg = gain margin (dB)
Phmarg = phase margin (deg)
Omegagn = normalized freq. defining gain margin (rad/sec/delt)
Omegaph = normalized freq. defining phase margin (rad/sec/delt).

Each of the above outputs is a 6x6 matrix covering all Ndelay elements and Mbox boxcar averagers. The number of delays increases going down a column and the number of boxcar averagers increases from left to right. Thus, the upper left corner is for 1 delay and 1 averager, and the lower right is the worst case with 6 of each. Note that when the system is unstable, having negative gain or phase margin, the resulting powers are erroneous. This method of integrating over a spectrum in the frequency domain to get a time mean square average assumes that the closed loop system is stable.

Appendix C contains the MatrixX simulation code necessary to compute the Greenwood power spectral density and for computing servo loop noise properties given a bandwidth, sample rate, ndelay and mbox.

Generally, the noise rejection criterion pushes the required servo bandwidth. Once a bandwidth is established, the sample rate is pushed by the requirement that phase margin be reasonable (near 70 deg) for some number of sample delays. Gain margin is uniformly good.

Figure 4 shows a representative cross section of the phase margin data. This was taken at a servo bandwidth of 7 Hz and 1 boxcar averaging step. The phase margin degrades with increasing number of delays and slower sampling rates cause faster degradation. If this same plot were drawn for a higher bandwidth, even faster degradation with ndelay would be shown.

Figure 5 shows how varying bandwidth filters perform subject to the more benign Greenwood noise model. The sharp breaks in performance at 2 delays for 15Hz bandwidth and at 4 delays for 10 Hz bandwidth signify the servo loops going unstable beyond these points. Note that the 5Hz bandwidth holds the residual power below 0.1 out to beyond 4 delays.

Figure 6 shows the same servo loop performance subject to the much more difficult Modified Greenwood noise model. It is interesting to note that the 10Hz bandwidth actually crosses over the 5Hz bandwidth performance soon after 3 sample delays. Perhaps contra-intuitively, the 5Hz servo loop rejects more noise than the 10Hz loop at 4 sample delays. This is a result of the low damping in the 10Hz loop amplifying noise near its cutoff frequency. Only the 15Hz servo loop meets specification of P_ratio<0.1, and then only for 1 sample delay, so the 0.01 sec sample rate implicit in Figure 6 is too slow.

Figure 7 shows the servo loop performance at 1Khz sampling for the Modified Greenwood noise model. The 10Hz bandwidth servo nearly makes the specification that P_ration be less than 0.1, however the more conservative 15Hz was chosen because it continues to make spec when mbox is increased.

Future Work

The most important thing to do is to decide on a noise model, since this has been shown to greatly affect the required servo bandwidth. It should be noted that there has been no effort to optimize the servo loop for some given number of sample delays and boxcar averaging steps. This means that the simple servo loop studied here is overly sensitive to sample delays and boxcar averaging steps. Considerable improvement in noise rejection probably can be had by building a better servo controller, for example one with an observer having many states to estimate the sample delays. This would also decrease the phase margin problems and probably permit slower sampling than the 1Khz found here.